Towards a quantum gravitational mechanics

A fantasia on the possible shape of a future theory

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Abstract

Whether space determines trajectories, or trajectories determine space, remains a subtle question post-GR. The modern distinction tends to be in terms of geometry and mass-energy, the tension between Einstein's 'marble and straw' - which is primary? In quantum theory the same dilemma, couched in terms of particles and fields, is turned into a principle of nature; the 'resolution' of the dilemma is to reify it. But everyone would agree that the conceptions of space and of 'object' are both logically and technically difficult ones in modern physics.

This article discusses the prospects for a more 'primitive' type of conception where this tension may in principle not arise, in which trajectories *are* space, or in which space is completely specified by a sum over all trajectories. The thrust of this approach is to construct, by a return to first principles, a space of basis states in which the traditional distinction between 'far-action' forces and modern relativistic 'field contact' forces can be subsumed. A particle- or wave-trajectory in such a space emerges with a radically different meaning. The trajectories live on a scale-free network from which space dimensionality and quantum mechanics (interpreted consistently with the Feynman-Wheeler advanced potential model and the Cramer transactional interpretation) are to be co-emergent.

Consistency with classical, relativistic and quantum mechanics is argued, and it is suggested that a novel conception of gravitation might be possible on the basis of such a non-field approach. The focus is on conceptual foundational issues, and we do not pretend to arrive at the basis of a proper theory. The objective is only to explore an alternative general framework that offers an intuitive interpretation of basic physical principles. We indicate where this framework makes contact with some existing avenues of approach to quantum gravity theory (such as causal dynamical triangulation, twistor theory and loop quantum gravity) and also where it fundamentally differs from any of these. We suggest, but do not attempt to prove, that it has potential for formal development.

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1.1. on the absolute & the relative in theories of motion

1.) The central question of **inertia** remains a mystery common to all generally coordinatised theories of motion. A brief background follows. As is well known, the simple Newtonian trajectory satisfying

$$\frac{d^2x}{dt^2} = 0 \tag{1}$$

contains an 'absolute' *acceleration* of a particle relative to an embedding frame of Euclidean 3-space, and all relative motions can be resolved into component absolute motions of this kind. This is the acceleration in Newton's Second Law, where

$$\overline{F} = m\left(\frac{d^2x}{dt^2}\right) = m\overline{a} \tag{2}$$

in general only because the acceleration is absolute. If $\overline{a}$ is defined as a *relative* acceleration then a more complicated force law supervenes involving other accelerations, due to 'fictitious' forces such as the centrifugal force, and the resultant generally is not in the direction of the 'real' force. In this respect, then, an absolute reference frame is an essential tool of the theory and works perfectly well (if one neglects an implicit violation of the Third Law by a non-reactive absolute space) to define absolute inertial mass.

2.) But an inference from the fact that there is one class of frames - inertial frames - which *does* satisfy Newton's laws for relative motions leads on from a restricted principle of Galilean relativity to a more radical theory of relativity (SR) in which the embedding 3-space is rejected in favour of an embedding semi-Euclidean spacetime. SR makes the inertial mass vary with the total relativistic energy; but there is still an invariant lower bound to the energy in every rest frame. Absolute rest and absolute velocity are removed; but there is still an absolute acceleration and absolute rotation relative to a privileged class of inertial trajectories.

3.) GR then proposes to remove all absolute motion by its principle of general covariance, and to replace local inertial frames (except over infinitesimal domains) by implementing Mach's principle in the field equations. But Mach's principle in its usual form assumes instantaneous far-actions between the 'fixed stars' and a local particle, whereas GR excludes far actions. Proposals have been made for an explanation of Machian inertia by implementing Weber's postulate that *the sum of all forces on a particle is zero in all coordinate frames.* Weber's force law

$$F_{1,2} = -H_g \frac{m_1 m_2}{r^2}\left[1 - \frac{6}{c^2}\left(\frac{\dot{r}^2}{2} - r\ddot{r}\right)\right] \tag{3}$$

modifies Newtonian gravitation by terms proportional to the relative velocity and acceleration. Here the inertia is a dynamical reaction force. Analyses by Assis (1993) and Assis & Graneau (1994,1995) show that the long-range $1/r$ force term proportional to the acceleration (the third term in eq.3) implements Mach's principle by effectively dividing the cosmic mass distribution into isotropic and anisotropic components. However all such schemes assume instantaneous far actions, and thus are inimical to conventional relativists on this account; whilst at the same time they appear to be radically classical in spirit, and are thus inimical to quantum theorists on another account.

4.) As it stands, the best theory of gravity available (GR) says that an inertial particle follows a timelike geodesic on a differentiable non-flat affine manifold instead of a straight line in Newtonian absolute space; but these geodesics pick out inertial trajectories against the background of a definite spacetime metric with a great deal of structure, and they define by exclusion a class of absolute motions. The boundary condition at infinity reproduces the function of Newtonian absolute space unless the field equations are modified to yield closed solutions, in which case they also yield solutions with structure in the absence of mass.

5.) So although GR is a better 'theory of gravity' than Newton's, and relates the general coordinates to the distribution of mass-energy, it doesn't tie the local coordinates to the global distribution of mass energy so as to successfully reclassify all rotations and accelerations as relative. It isn't necessary in this theory that all inertial geodesics are occupied states, or are states directly anchored to mass particles, so that spacetime does not (as Einstein had hoped it would) reduce to a conventional system of reference for registering coincidences of material points. The theory contains empty spacetime solutions for gravitational waves independent of the presence of mass. Thus it still contains absolute motion, and it doesn't explain it.

6.) One seemingly unconventional response to this problem is to accept it as part of the solution, allowing some property which has a kinematic transform as 'absolute motion' to enter as a primitive attribute of the elementary 'objects' in the theory, making an internal degree of freedom *prior to* any conception of generalised coordinates. This has been shown to be a legitimate relationist position (*cf.* Sklar, 1976; Friedman, 1983) and in the trajectory-centred approach explored here it will emerge as equivalent to saying that what we call 'intrinsic mass' is actually a result of a non-zero lower bound on potential energy, or that *all* mass-energy including 'intrinsic' mass is actually extrinsic, i.e. structural.[3]

7.) The distinction between a Newtonian *vis insita* and the above primitive attribute is that the former is a quantity attached to a point object reacting against an absolute space, whereas in order to achieve background-independence the latter must incorporate the functions *both* of 'space' *and* of 'mass' in the form of a dynamical equilibrium internal to some elementary object. In the scheme proposed here, the scalar mass and coordinate-space functions of a system of 'particle trajectories' are jointly replaced by transformations of a system of primitive unit vectors. To realise Mach's principle as a lower bound on these transformations we require this underlying unit vector to represent a connection between measurements which is based *neither* on finite-speed relativistic 'field contact' forces *nor* on infinite-speed Newtonian 'far action' forces. This is just the type of connection - a 'nonlocal correlation' - which occurs when SR and quantum theory are brought together.

8.) This leads to the idea of a system of anisotropic dynamical objects replacing the manifold of 'free' affine trajectories, embodying space structure through scale-free neo-Machian 'nonlocal contact forces'. Spacetime structure is to be emergent in the end-to-end graphical self-interaction of a system of such unit objects, and the underlying nonlocality identifies an embedded critical-point graph phase of effective *reduced dimensionality,* analogous to that of a 2D quantum film where the lattice correlation length goes everywhere to unity.

9.) The emergence of 4-space in this scheme will be seen to have some connection to quantum

3 Origin of intrinsic mass is conventionally sought by bolting Higgs fields onto a supersymmetric Standard Model (SSM). The perspective explored here will suggest that the requirements for Higgs and for supersymmetric partners in the context of the SM might, instead, both stem from a non-point unit object in the state space of a quantum gravity theory underlying SM and GR. This linear object would have dual local spacetime representations as a zero-point minimum vacuum potential for a pair of fermions *and* as a bosonic coupling, in a form of contextual or dynamical supersymmetry. Thus far there are obvious parallels with the superstring programme positing non-point primitive objects near the Planck scale. But our proposal does not live at this scale, or at any particular energy scale, and is only emergently a local field theory. We propose instead that spacetime *trajectories* would live on a *complete graph* of N such nonlocal unit objects, whose mutal constraint (under a certain exclusionary condition to be described) would supply a renormed zero-point of inertial mass across $N^{1/2}$ displaced graph vertices. The underlying nonlocal state space could be thought of as a scale-free lattice of unit objects in a critical-point phase where correlation distance is unity everywhere, whilst the local structure of self-interaction is emergent *via* a renormalisation flow that carries the critical point over *all* real scales and breaks the meta*s*ymmetry of *doublet basis states* to singlet local states. The nonlocally-coupled ends of N objects thus form dominant local couplings at $N^{1/2}$ vertices of the graph (or from another point of view a single nonlocal object breaks to N locally-coupled instantiations of itself), and only a trace of the underlying metasymmetry survives decoherence in entanglement correlations. (see Sections 2.4, & 2.5).

gravity theories based on the principle of causal dynamical triangulation (CDT); but crucially, unlike CDT, which is defined near the Planck scale in accordance with traditional quantum-gravitational prescriptions, our proposal is that each instantiation of the nonlocal 2D triangulation operation carries the primitive reduced-dimensional quantum phase into spacetime over *all* scales and at all epochs.

10.) In developing this idea the objective will be to avoid the introduction of empty states in the first instance. By justifying the idea that spatial volume is an emergent *effective* property of richly complex interacting systems of fundamental entities we eliminate volume from the toolkit of basic principles and move radically away from the conception of the classical central field. The conception of the classical central field is of a spherically symmetric infinite volume swept out by an infinite series of concentric smooth wavefronts, each defined by an infinite number of point states, any number of which may remain empty states to a radius of infinity. There is almost nothing intelligible about such a structure. Some modern semi-classical and quantum theories of electrodynamics have attempted to regain intelligibility by eliminating the degrees of freedom of the field.[4] We propose to eliminate extraneous degrees of freedom of the spatial volume in an analogous way but at a more primitive level of the ontology, in such a way that the meaning of ‘absolute’ and ‘relative’ states will change dramatically. It is not even clear that there remains any kind of meaningful distinction. This could be construed as extending the powerful notion of complementarity to the traditional distinction between ‘object’ and ‘space’ (‘straw’ and ‘marble’), leading on to the observation that in place of the central field we are proposing a radically discontinuous kinematic and dynamical space, and that there is something naturally congenial to the spirit of quantum theory in such an approach.

4 In particular, the Wheeler-Feynman absorber theory of electrodynamics, developed by Hoyle & Narlikar, Davies, Cramer and others. This programme becomes very relevant later on.

1.2 scope & motivation of the present work

1.) To set out the motivation for considering a primitive trajectory-based approach we can begin by addressing the question: Why is such a description not a mere tautology? An exhaustive register of all real particle trajectories for all time would contain a full description of spacetime for any measurement purpose. But no useful theory can invoke such a register. A useful theory has to be an imperfect representation of the world, just because the complete description is not locally available to us. Obviously the meaning of the fact that for us, at any given here and now, a useful theory has to be *predictive* is that so much significant information is locked away in 'future' states. Further, it seems reasonable to state that the existence of a limit to predictability here and now is not of itself the *cause* of these future states being inaccessible: the quality of predictiveness belongs to the act of theorising; the inaccessibility is a given fact of the ontology.

2.) The origin of the light cone runs ahead of past-timelike states to depict a local order, but is *embedded in* the complete causal structure. The requirement for self-consistency (expressed in the criterion of predictiveness) demands that somehow the apex of the here and now has to be representative of the information in future states as well as of information in past states. That future states are empty states, waiting to be filled by the outcomes generated by past states, is not at all clear. The past-timelike half of the causal structure is known to be insufficient on its own to determine quantum states; only a sum over all *possible* histories suffices to yield even a probability here and now. But presumably the sum of the probabilities of all cosmic events must in the end total to exactly unity. This suggests that the causal structure as a whole does not care about the philosophical distinction between determinism and teleology insofar as that distinction is concerned with the sign of the time variable. The universe in some sense just 'is', consistent with the idea that what is called the 'microscopic reversibility' of classical and quantum processes is an essential symmetry, which gets hidden in complex thermodynamic systems.

3.) If the thermodynamical 'breaking' of time-symmetry is an emergent feature of large systems[5], whilst the underlying causal structure remains symmetrical, then it appears that a 'complete' theory must be able to show how a state at the here and now represents in some sense a resultant of contributions of all particle trajectories taken over all past and future times. Of course if this is conceptually correct then it becomes true by definition that a predictive formalism can never be more than an *emulation* of a complete theory, since the complete theory would be a botanical catalogue. But a theory need not be calculable to all orders for it to be physically non-trivial. The

5 We stipulate, crucially, that this means systems of large *quantum number*, not primarily systems of large scale.

model of QED suggests that a promise of statistical predictability in some limited problems would suffice to identify it as a possibly useful theory (after all, GR cannot be solved realistically for the vast majority of physical interactions).

4.) But surely a trajectory-based summation *is* merely tautological, even if incomplete and statistically predictive, unless it can show why the sum of all spacetime states of particles is not just the same as a sum over all other field quantities. For this, surely, is the function of the differentiable spacetime manifold whose infinity of empty point states we are proposing to replace, the mechanism that will make the theory a 'theory of gravity' in the sense of GR? The GR metric is exactly the sort of generalisation that one needs in order to get from a merely redundant description of trajectories to an *interpretation* of the correlation between mass and spacetime displacements which those trajectories embody.

5.) This is all true, but problematical. A sum over the potentials of all other field quantities is required by the theory to be *exactly proportional* to the gravitational potential, since this sum is just the energy-momentum tensor which determines the metric tensor. But the fact that this proportionality is *not* a perfect isomorphism is a mystery. The metric tensor is something coupled with, but not found 'inside of', the particles themselves; it has autonomous components. That is, the components of the metric tensor do not all vanish even when the energy-momentum tensor is zero, i.e. when there are no mass-energy trajectories in it. In this respect the theory has an awkward unspecified degree of freedom even in its minimum condition. True, the energy of a quantum field is required always to be an average of fluctuations around zero; but this is a poisoned chalice because quantum field theory then predicts that the vacuum energy should destroy the universe, exactly on account of the fact that the spacetime manifold is a differentiable continuum of single-valued position states.

6.) It may be that GR is not the right generalisation. On the other hand, Einstein's programme to make the gravitational/inertial 'forces' completely redundant inside a theory which *identifies* spacetime with the mass-energy distribution would be fulfilled by a non-trivial description of spacetime in terms of a geometry of non-differentiable (in the limit) trajectories. Eliminating undesirable components of the spacetime structure might also hold out the prospect of cancelling an unwanted vacuum energy.[6] Is this possible in principle?

7.) Since quantum theory requires all fields to be associated with particles there is an equivalent

6 Of course we would have to explain how our theory accounts for theoretically useful vacuum contributions like the Lamb shift and vacuum polarisation.

description in terms of trajectories, but not a reduction to them, since the trajectories are only particle-like in specific acts of measurement. In 'between' measurements the path of a particle cannot be determined instant by instant, so the trajectories are wave trajectories; momenta are carried by the field. This means that if trajectories are to be recovered from quantum field theory as primary entities, then plainly this cannot be done in terms of classical-deterministic particles because these are not adequate to support the whole causal structure.

8.) This all points, we will suggest, to a scheme in which the root distinction between fermion and boson statistics arises from the distinction between nodes and edges (connections) in a complete graph network. The bosonic phase of the emergent local spacetime structure (null photon signal lines), and its fermionic phase of coupled pairs of position states, can be seen as complementary aspects of a nonlocal *doublet position basis-state* in the underlying state space of the theory. And in this way we will lose the idea of continuity underlying the concepts of both field and point-particle, and arrive instead at the requirement for a non-zero minimum path (in the underlying state space of the theory) identifiable with - from one point of view - the existence of breaks of direction in a continuous self-intersecting curve, thus associating a non-zero minimum acceleration with the included angle of a dynamical triangulation measurement.

9.) The rest of this article considers some of the detailed issues raised by this proposal, leading to some suggestions for the implementation of an unconventional mechanism for the gravitational action. So as to identify it, the framework is called parcellular mechanics or PM.[7]

10.) Meanwhile we can sum up some desiderata of the approach as follows: The world is to arise in the self-interaction of a system of non-differentiable causally time-symmetric trajectories. Each possible trajectory is to be somehow equivalent to a sum over all others in a path-integral approach to a finite perturbative theory of 'quantum gravity' where *a*) the trick would be to define 'all possible' so as to restrict the sum to a finite number of filled states and eliminate the degrees of freedom associated with empty states, and *b*) the register of all states needs to act nonlocally to determine each local trajectory.

11.) Traditionally most effort focuses on trying to reproduce GR as a quantum field theory, but if GR is a classical approximation to some underlying quantum theory it is not necessarily the case that the underlying theory, even if correctly formulated, would ever reproduce the calculability of GR for astrophysical problems, any more than GR is a calculable theory for particle interactions.

7 The origin and meaning of the term is of no importance here.

Neither is it at all to be expected, from this point of view, that the underlying theory need ‘operate on’ any of the elements that are part of the machinery of GR. In particular, an underlying quantum theory of gravity might be the *begetter* in common of the phenomenology modelled in GR *and* QM, yet be qualitatively different from either. The point I wish to emphasise is the corollary that quantising the gravitational ‘field’ (or the metric tensor) is not necessarily the route to this theory, meaning that the dimension of quantisation in the underlying theory, and the length scale(s) associated with quanta, are not necessarily those generated for the purpose by applying quantum theory to the spacetime of GR.[8]

8 String theories abide by that application and therefore live at very small length scales. They also seek to preserve quantum field theory unchanged and operate against a continuous background space. The point of view explored here is probably closer, in spirit, to the twistor programme, though no attempt is made to demonstrate this.

2.1. basic philosophy of the state space

1.) This section will inevitably be a bit abstract. A good place to anchor the discussion is **Newton's First Law**: *A body continues uniformly at rest or in motion in a straight line unless acted upon by other forces.*

$$\overline{R} = \sum \overline{F} = 0 \tag{3}$$

The point of stating this law here is not to begin discussing classical mechanics (see *Section 2.2*) but to remind ourselves of how much is assumed in the simplest of principles. None of the several terms in the verbal unpacking of the first law have meanings that are known *a priori*. Therefore we propose that definitions of 'body', 'rest', 'motion', 'straight', 'line' and 'force' begin operationally with a '**measurement**' that assumes nothing about space properties. A measurement is defined to be an operation performed by the system of nature on itself, an operation that generalises to a **register** of a change in some ratio of quantities (characterised usually by kinematical and dynamical states of 'particles'). Sets of such operations are the correlatives of 'observations', but this implies no preferential status for an 'observer', and nothing is known *a priori* about the mechanism of observation or its correlatives. The system of nature is to be defined as the aggregate register of all such **operational ratios** arising in the self-interaction of the system.[9]

2.) Given simply that 'things happen' it is not hard to justify assuming a large number of such self-interaction states (i.e. nature is a plural system, not an undifferentiated plenum), and it seems axiomatic that these states must be systematically distinguished from one another, and systematically related to one another, which in turn leads to the requirement for a **space** of states. As nothing is known about the nature of this space it can be introduced with arbitrary dimension and no definition of distance. All that is primitively necessary is that registration of discrete states is possible, and so in order to import a minimum of *a priori* assumptions we will assign **each** registered state to a distinct **orthogonal dimension**.

3.) We assume that changes in ratios of quantities must be registered self-consistently on all dimensions, for which purpose the dimensions must intersect one another according to some

9 Note that there is no explicit discussion of the quantisation condition for the system at this stage. From a foundational point of view we regard the need for, and implementation of, such a condition as having yet to be discovered.

scheme in order to represent **causal contiguity**.[10] Next, if we acknowledge that it is found experimentally and theoretically convenient to identify the registration of states with relative displacements of 'particles' (which we are careful not to define too closely just yet), then we would require a **position space** of *n* dimensions each representing one of *n* particle states. Each dimension then acts as the operationally-defined locus of one particle in the space and registers thereby some ratio of quantities which will define its state relative to other states on other dimensions.[11]

4.) Now what is the minimal additional structure necessary to permit the dimensions of the state space to represent a range of ratios of quantities? The answer to this question leads us to a nice ambiguity.

- We start with the condition that each locus should be **bounded**, since a ratio of infinite quantities makes no mathematical sense and a 'particle location' of infinite extent promises to make scant physical sense. (Fundamentally it is the *operational* sense of a system of 'measurements' that constrains our theory to include only finite ratios.)
- We have assigned one dimension per state, so the minimal degree of freedom of each state will be a **straight line**.[12] This means a line which is determined, like an ordinary Euclidean straight line, by two points; but, *unlike* a Euclidean straight line, we stipulate that our line is determined by ***only* two points**.
- This very strict definition imposes **straightness** in a more fundamental sense than the

10 This may sound like a superfluous condition. Lorentz invariance requires that all the axes of 4-space intersect one another at *every point* of space and time equivalently and one would expect to generalise this to any *n*-space. How could an intelligible causal structure arise if degrees of freedom *x, y . . . n* were not available simultaneously to each 'particle' and to all particles equivalently? GR tilts the light cones on the tangent spaces and deforms the manifold, but even GR leaves Lorentz invariance intact at every infinitesimal point. How could one coordinatise a (hyper)space where only some fraction of all dimensions directly intersect one another? Even in string theories the compactified dimensions are only inaccessible on large scales and are available equivalently at every 'point' of spacetime down at the string scale. But consider in parallel the orthonormal basis of the infinite-dimensional vector space of quantum theory: Every state vector evolves linearly and causally remote from all others until the point of reduction, so we get the idea of *n* degrees of freedom in a one-to-one correspondence with *n* independent processes. Obviously the Hilbert space has a lot of machinery and functions that we don't know about yet and we are not attempting to reproduce it. But it's worth pointing out that we do not know *a priori* either that there are an infinite number of position states, or that they are *a priori* interchangeable, or even that the notion of 'position' has a singular meaning; so it is not certain that it is ever valid for vectors associated with a minimally-defined 'point' in a our space of position states to evolve from one common origin. Strange as this may sound, the fact is that the meaning of position is intimately bound up with the meaning of gravitation, and it is in this area that quantum theory is troubled. So in implementing the principle described in Para. 2 we will bear in mind that it might not be possible to construct a viable position space for a PM 'particle' where all state vectors pass through a single origin. Whether or not a many-origin space would be useful - i.e., if it suggests a conceptual and formal connection both with GR spacetime and with the quantum vector space - remains to be seen.

11 There are of course more degrees of freedom per particle in conventional physics than just position, but this is in a sense a statement of the problem we wish to solve: What is the general scheme of which these freedoms are a particular case? We justify the simplification at the outset by pointing out that the kinematical state of any fermion in a set of fermions indexes the contributions of all field potentials of the set through its coupling to the gravitational potential.

12 There are, we know, special geometries in which finite geodesics are unbounded, but these involve postulates in higher-level spaces that are not generated directly in our state space. For example, the violation of the Euclidean parallel postulate in Riemannian space produces a finite and unbounded geodesic line as a closed curve. But our construction is very general and says nothing about any higher-level geometry of the line; we would like it to remain general.

Euclidean definition; that is to say, we decree that there is no fuller definition in principle of the intrinsic state of the line than the state of its two determining points, or in other words its geometry has the minimum possible information content.[13] (This does not exclude the possibility that the line acquires *extrinsic* states in different 'observer' representations.)

- The same definition imposes **boundedness**, because if we are to be able to determine it by visiting two points they must lie a finite distance from one another[14] and since we know there are *only* two points, these points, once located, are by definition termini of the line for all possible measurement operations.
- Moreover, where a Euclidean straight line may be divided into two 'rays' or parts of itself by an arbitrary point anywhere on its length, our line is by definition **not divisible** since interpolation of a point generates *two* lines under the above definition, not two halves of a single line, because the guarantee of straightness now applies to each separately.
- Finally, if two points exhaustively define a line then it follows that *any* two points must be connected by a line under the above conditions. This places a further very important constraint on the construction: *Each boundary of every line must be connected by a line to both boundaries of every other line* - or in other words n linear dimensions represents (for large n) the connectivity of a **complete graph** of approximately $n^{1/2}$ boundary points.

5.) Perhaps the definition in Para. 4 looks less like a minimal definition of a line than like a definition of no line at all? This is so, in the sense that the ontological status of the line is that of any spacetime geodesic line, i.e. an **empty state** that represents a *possible* physical state. But we now want to make the line more concrete than any affine geodesic by requiring that that it *always* be a **filled state**. Functionally, if one line, or dimension, represents uniquely the 'position' of one particle in the state space, then each such position is a unique 'point' or each particle state is **isomorphic to a volume element of a linear state space**, and the definition of a measurement of position in the state space can be nothing other than a set of relations among 'points' of **different linear scales**. Infinitesimal Euclidean points are irrational in this space (schematically, Fig. 2*a*) because they are **degenerate** states. Two intersecting orthogonal dimensions of infinitesimal scale are infinitesimally separated, and they cannot be related by a real line. Their co-location robs them of the capacity to represent ratios of quantities (Fig 2*b*). We could say that they are, conversely, separated by an

13 Note that this 2-point line will by definition always rotate onto itself in a congruent transformation, which completes the analogy with a Euclidean line. We don't want to place stress on this particular point, however, as we wish later to complicate things slightly by incorporating an 'intrinsic spin' symmetry which means that only *one* of two rotational degrees of freedom will allow a congruent transformation.

14 Operationally and geometrically speaking, *finitude ≡ triangulability* because an infinitely separated pair of Euclidean points subtends 180° from any measurement point, and a 3rd collinear point is forbidden by our definition of straightness. This represents our accommodation with the Parallel Postulate at the cost of losing metrical infinity, whereas an indefinite number of possible measurement points on unbounded parallel-displaced geodesics in GR represents violation of the Postulate and the preservation of metrical infinity.

imaginary line; but this is just another way of saying that their conjunction reduces to a nonlocal *identity,* from which we still need to recover a local correlation of a pair in order to satisfy the definitions of straightness, boundedness and indivisibility in Para.4. Notice that we can perform this analysis beginning with either dimension and arrive at exactly the same degeneracy in the position states of the system. They collapse, leading to equivalent definitions of an identity (Fig 2*c*). Removing this degeneracy requires, apparently paradoxically, an expansion from the singular space of two collocated points to the space of two coordinate lines, thereby exchanging singular definite Euclidean points for **doubly-connected position states** each with an internal degree of freedom. We therefore find that two such doubly-connected point-elements of PM space *intersect at* a Euclidean point in emergent 3-space. (This curious inversion perhaps suggests to us that 3-space Euclidean points behave like *defects* in the space of PM position states.)[15]

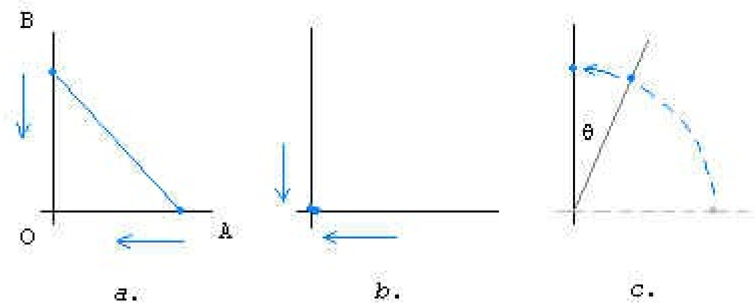

Fig. 2. Schematic suggestion of the degeneracy of point states

6.) It seems that we are led to accept this strange property of PM state space which means that we cannot have a non-degenerate notion of 'position' which has both the *functionality* of a point and the definition of a point in Euclidean geometry. This actually turns out to be useful. A Euclidean point is a mathematical orphan that has no viable state except as it is adopted by some imposed metrical gauge, and the reason is that it is (so to speak) an 'only twin'.[16] Laying down sheets of coordinates after the fact, to allow the point to be given a Euclidean representation, is exactly what loses the essence of the functionality which the point has in our representation, and some

15 A possible analogy has already been hinted at (*note 6*) between the element 'unit scale' and the unit vector in an adapted Hilbert space. We can perhaps begin to see how *n* position vectors each representing a trajectory in PM space might rotationally transform onto one another at a common origin corresponding to a point in 3-space. But one major adaptation, we now confirm, is that the PM position state is not determinate for the set of radius vectors that intersect any *single* origin, for the reason that it is itself always congruent with one of these vectors. It has *two* origins, and represents, we will say, two antiparallel moduli which correspond to arbitrary complex arguments. In other words there is a fundamental duality in the PM construction that involves, instead of singular position states, conjugate pairs of (as it were) *half-position* states which are co-dependent in the sense of the two ends of a string.

16 Like the immortal Peter Cook, 'tragically, [it] was born an only twin.'

functionality then has to be reintroduced by making the 0-D point a differentiable function of an *n*-D continuum, with the result that **infinities** occur. Instead we wish to preserve our more general definition of a point in state space.[17]

7.) In short, the relation of two particle states must be representable as a relation of two lines primitively; we cannot begin with point states of zero extension but must include extension radically in our definition of '**position**' in the state space. This means that 'position' is never a single-valued function; rather, one 'position' enters as a function of another 'position', both of which involve **pairs of boundary conditions**. In this way 'distance on a line' enters in the first order as a mechanism for registering a ratio of quantities (inside elementary *triads*, as we will see), and a single-valued point-position can only be projectively recovered as an extremal *idealised* case.[18]

8.) So, if there is no strong *a priori* motivation for doing so, we do not propose attempting to reduce this redundancy. **We accept that a rotation of points into lines and lines into points is an elementary symmetry** in PM space. This is extremely useful because it allows us to resolve what would otherwise be the paradox of an infinite number of points of zero-dimension redundantly defining each of an infinite number of empty position states (which is to define a continuum singularity). The solution will be that points of dimension 0 have, irreducibly, equivalent projections as lines of dimension 1, because **points do not occur other than as the termini (i.e., in general the vertical conjunctions) of lines**; conversely **every conjunction of lines, and *only* a conjunction of lines, defines a unique point**. The result of this construction, given a PM space of *n* volume elements (see Para. 4 above), is that each such unique point in 3-space 'contains' roughly $n^{1/2}$ lines (or what we might call 'virtual' degrees of freedom).

9.) The above considerations show that the natural framework for PM space is given by the axioms

17 We can note at this stage that one way of expressing a redundant pairing of collocated points would be in terms of null lines. Such lines occur in a relativistic spacetime geometry as the signal lines of photons. It is possible to construct a geometry of purely null lines, but obviously some important property of a line gets left out in such a geometry - i.e., real length, whose physical analogue is an ensemble property called relativistic mass. It is a consequence of Lorenz invariance in 4-space that any system of intersecting null lines is automatically a system of orthogonal lines. Consider an arbitrary system of photons: Infinite Lorentz contraction on the proper 'motion' axis of each null signal line reduces away all parallel components of all intersecting null 4-vectors connecting it with the origins of other photons (or, the system of electrons is contracted to a plane normal to the photon spin axis), leaving only 1-space transverse components. Thus, every photon null line intersects all other photon null lines orthogonally. Since only two orthogonal lines define a plane in two dimensions it is obvious that the system of *n* photon lines where $n > 2$ is contracted onto a 'plane' of *n* dimensions. We will show that in PM this hyperplane behaves 2-dimensionally in the sense of a critical-point system whose correlation length is always equal to the confinement distance.

18 The original motivation for 'parcellular' mechanics was Schrödinger's argument that classical mechanics had never been deterministic in the first place, since velocity is required as an initial condition but cannot be defined in an 'instant'. The concept of instantaneous velocity is a theoretical abstraction from actual processes, which are minimally pairs of interactions to which an interval of time is innate.

of **projective geometry**, which assert a complete duality between points and lines, such that:

i) for any two points there is one unique line intersecting them both; or

ii) for any two lines there is one unique point that is the intersection of them both.

A projective 3-space built from such point/line elements has the properties that

iii) each point is also a line through an origin in the 3-space, and

iv) each line in the projective plane is a family of such lines, including a line 'at infinity'.

This is a non-Euclidean geometry of the greatest possible generality.[19] We will try to preserve a definition of a line or point which retains this generality, and avoid a field of continuous background coordinates. The elementary unit of point-line relation under this definition will be 'unit scale', for which the first real values will be generated trigonometrically in triads, extending over a complete graph of triads of intervals transforming under the Lorentz invariance group. We assert that it will be possible to recover the Riemann general analytical transformation for continuous geometries of *n* dimensions from the type of projective geometry envisioned here, and in particular that geodesic displacements on the curved 4-space manifold of GR can be shown to be dual with the more primitive Cayley-Klein type representation of distance in terms of projections between points and lines.

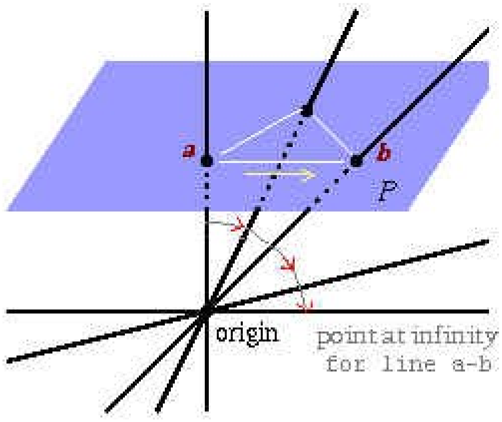

Fig. 3. Projective geometry. A line ab in projective plane P bounded by limiting lines through an origin lying on a line at infinity in 3-space. A sheaf of three boundaries generates a triangle on P.

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19 A similarity to the projective twistor correspondence is evident here.

10.) The conservative definition of Para.4 ensures that there are no degrees of freedom for the line (or point) other than those determined by other points (or lines). In other words an assembly or **network** of lines self-sufficiently defines and exhaustively fills the n-dimensional 'pseudo-volume' of its own space; there is no embedding space from which a line or point may borrow infinite degrees of freedom. We have obtained this by allowing each linear element of unit scale to define one orthogonal dimension and requiring it to be a filled state. (Aside from conserving the generality of the projective axioms, this may seem at first not to be physically conservative despite *Footnote 4*; but for now we leave this procedure to be justified by its results.) A number n of exemplars of linear unit scale can then be regarded as analogous to the n volume elements of a PM space of arbitrarily high dimension n.

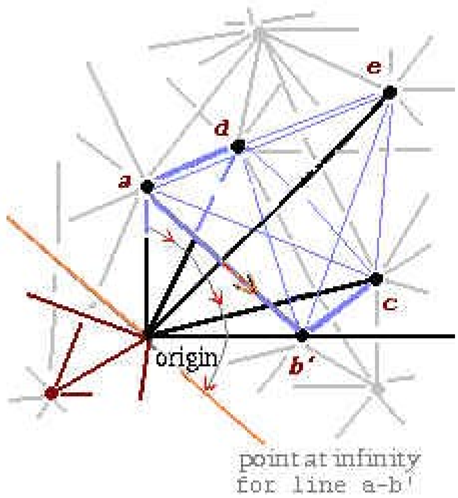

Fig. 4. A generalisation from Fig.1. Lines whose boundaries are generated from any common origin lie on arbitrary numbers of different planes in projective 3-space.

11.) The **boundary condition** in this space is neither at infinity nor at any definite real 'radius'. The n volume elements are all bounded and dimensionally orthogonal, and realise an arbitrary spectrum of real distances. *This is not a space that has any well-defined characteristic scale.* Every point (of 'measurement' or self-interaction) is a distinct projective origin, and there will n pairs of boundary conditions on n lines, each pair projecting to approximately $n^{1/2}$ 'points at infinity', with each 'point at infinity' being a common boundary condition on n other lines. We can think of the boundary condition as having been distributed or *dispersed* (in a process of 'virtual partition', see *Section 2.5*) through the body of PM space so that every one of $n^{1/2}$ boundary nodes is a new point at infinity, an origin for a 'new' projection of lines in a vector-space which (for finite n) always remains closed (see Fig. 4).[20] A related conservative assumption will be that changes occur only in the form of registrations that can be self-consistently absorbed into the state of the whole network of lines and points without the associated quantity going to infinity.[21]

12.) PM space will have many **distinct boundaries** or peripheries, each also a distinct **projective origin** or centre. This may seem paradoxical but physically speaking it is highly desirable, and consistent with the principle of relativity. Interpreted in the light of quantum theory, the deep meaning of GR is that space is a structure composed of many views of itself, all of which are reconcilable but none of which are quite equivalent. Like the so-called holographic theories, PM realises this condition. Because it is a process of self-interaction, not an object, PM space is a register of *operations*, and the sequencing of these operations displaced in space and time is the obverse of a simultaneous collocation of coordinate origins. In other words we cannot treat any actual origin (point of measurement) as simply reproducing the function of a single imaginary origin, and we cannot treat any single operation as an equivalent scaled down copy of the sum of all such operations. It is therefore never valid to map any number of original operations onto the same set of simultaneous co-original Cartesian coordinates.[22]

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20 This ramifying network could be thought of as analogous to the Huygens construction mapped to an arbitrary number of projective planes, where an infinite number of wave normals is replaced by a finite number of discrete 'rays'. With the exhaustive connectivity of a complete graph, each junction renormalises phase at a new projective origin for the *entire network*.

21 This constraint on the graph - expressed also in the PM 'exclusion principle' which, as will be shown, generalises to require the non-degeneracy of *lines* (i.e. pairs of 3-space points) - implies the absence of *multiple edges* and *loops* - a loop being the exclusive connection of a point with itself. The absence of self-connected loops is equivalent to eliminating the self-energy of an electron's self-interaction with the field and 'multiple edges' only occur in the sense of states wholly separated 'in time'. In other words the PM 'supersymmetric' exclusion principle acts on lines so as to demand that 'multiple-edges' are not parallel-occupancy states but are *serial*-occupancy states, i.e. they define a *sequence* where an angular frequency associated with a periodic stationary condition defines a local 'clock rate' or time displacement rate. Expressed as the bosonic state of the PM unit-object this becomes a photon exchange rate, which quantifies the charge coupling constant. I wish only to note in passing at this stage that these constraints produce the *logical structure* of the canonical Einstein quantisation condition for radiation: That one photon from one electron goes wholly and uniquely to one other electron. The conserved quantity associated with time displacement symmetry is energy, and in general we would expect the *linear interaction* condition of our finite graph to produce the characteristic energy spectrum of a finite-state 'cavity'. From this networked cavity of linear stationary states we hope to pass in a direct way to a model of quantised oscillators from which the Planck radiation theory can be recovered.

22 This doesn't mean that such a mapping to co-original coordinates cannot be done; only that the map will fail to disclose the underlying divergent causal structure and, used as a guide to new theories, will mislead. The tension inherent in such mappings can be seen in SR, where Minkowski geometry allows one operation to transform onto another according as different observers choose different spacetime coordinate axes. Insofar as all lightcones are parts of the same flat lightlike hypersurface this is still a continuum approach, which fights against quantum theory, but an idea of the distinctness of inertial observer-frames is contained in the fact that transformations are now no longer simple Galilean translations which, if the coordinates were rescaled, might be superimposed, but are instead relative rotations that break (though continuously) the simultaneity condition of the Cartesian symmetry. In GR the lightcones are tilted and this tension is extended to a distortion of the manifold itself, associated with individual mass-energies; but the twisted 4-space map remains unbroken and so the Cartesian positional degeneracy is preserved. The paradigm of such degeneracy is of course the Big Bang singularity. In PM, spacetime singularities and matter singularities are both forbidden by the same dualising 'exclusion principle' which states that no two mappings of PM space onto any two points can be co-original mappings - they are always boundaries on a line in 4-space.

13.) In PM space it is logically necessary that each elementary measurement operation, where a point operates on the boundary conditions of a line, defines a **plane triangle**. But this will not be a Euclidean triangle in the real number plane, where two degrees of freedom suffice for the components, x and y, of one real-valued vector argument; because a plane Euclidean triangle of real vectors belongs to the space of synchronous Cartesian coordinates and is governed by the group of Galilean transformations. We know that Galilean relativity has to be subsumed by the Lorentz group, which can be represented as a rotation on a flat affine Minkowski 4-manifold. But SR does not 'include gravity', and in any case our requirements rule out the differentiable manifold, so we look for a different representation of the Lorentz symmetry. A non-Euclidean space of real vector gradients (tensor space) as in GR does 'include gravity' by confining Lorentzian symmetry to an infinite number of infinitesimal domains, but this will not do for us either, because the tensor space preserves the affine connection through a continuous transformation. GR implies the need for a many-centred structure but does not really supply the means.

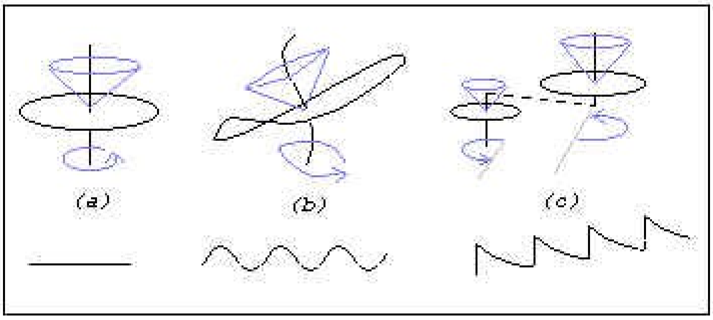

Fig.5. Schematic idea of discontinuous transformation of Lorentz symmetry in PM.
(a) SR, continuous flat affine rotation. (b) GR, continuous non-flat affine rotation. (c) PM, discontinuous non-affine rotation.

14.) Instead of distorting the SR manifold continuously we need to break the SR symmetry *dis*continuously somehow. And because the general double-connection of position states in our theory is prior to metrical scale it is evident that the symmetry breaking phase must also occur throughout the network independently of distance scale. This involves breaking the symmetry of the group of affine transformations which is fundamental to conventional metrical space, abandoning the idea of parallel displacement of infinitely-near covariant field vectors in favour of a scale-invariant type of transformation between the different and changing 'celestial spheres' as it were 'seen' from each of many distinct projective origins. The type of transformation suited to this structure is a multi-centred rotation which breaks the global continuity of the affine 4-manifold and so, by definition, will not offer a smooth transformation between all views (Fig.5).[23] Making such an idea work depends on the idea that each displaced view of the world sums over many distinct representations ‘simultaneously’ with actuality emerging in the sum of all (complex) views.

15.) Each of these projective origins (corresponding to a point of measurement) has to allow the system of nature to reflect itself there by generating a unique system of coordinates, each encoding one (dynamical and kinematical) view of the whole, such that the superposition of all such states reconstructs (in imaginary time) ‘the’ state of the whole. Such a space is plainly nonlocal in some degree, but we take it to be beyond argument that this is a requirement of any modern fundamental theory.[24] At this point we notice once again that the type of structure we are moving towards appears to have a more natural affinity with the structure of quantum mechanics than does the continuous differentiable manifold.

16.) To implement this idea in a toy form we suggest that a trigonometry of vectors in a ‘stack’ of complex planes has a rich enough internal symmetry to associate each distinct operation - that is, each distinct *vertex* - with a *unique* centre of rotation in PM space, and propose that oscillating states generated within the superposition of complex vector arguments encoded at each vertex can represent the quantum wavefunction. This strategy shows an obvious affinity with the complex rotational invariance used in the gauge theory of electromagnetism and will hopefully allow us to understand (when generalised) how a huge but finite number of superposed Lorentzian domains, each associated to a privately coordinatised complex hyperplane and displaced by some ‘hidden’ fraction of phase, goes over to GR continuity on the one hand, and on the other hand deconstructs to a projective space of quantised linear actions.

23 The point is that this is a topological procedure at a more primitive level than the transformations of relativity theory.

24 Context-dependent intrinsic spin parameters, Aharonov-Bohm type effects, EPR correlations in general, and nonlocal energy in GR itself - not to mention the problem of inertia - are all major reasons for saying this.

2.2. consistency with classical mechanics

1.) Now we are ready to return to consider the implementation of **Newton's first law**, *A body continues uniformly at rest or in motion in a straight line unless acted upon by other forces.* This law for a body in inertial equilibrium

$$\overline{R} = \sum \overline{F} = 0 \tag{4}$$

is a limit of the **Second Law**, which states that *the rate of change of momentum is proportional to and in the same direction as the force*. Both are satisfied by a classical 'particle' of any size because of the spherical space symmetry and Newton's 'mass point' lemma for central forces; but insofar as the Second Law might be applied to a 'particle' in PM, the First Law is not reached as a limit of it. This can be interpreted as saying that the spherical symmetry of central forces does not hold in PM. The volume element which is the analogue of a 'point' in PM space has a *directed internal degree of freedom*[25] given by the complex projective identity of point and line such that a particle-analogue isomorphically identified with it cannot occupy a position of scalar equilibrium. That is to say the interval between any two points of measurement reduces universally to a limit in which

$$\overline{R} = \sum \overline{F} = m\overline{a} \neq 0 \tag{5}$$

because taking the time derivative of velocity to $dt = 0$ in Newton's continuous space is not a fundamental operation in PM space according to the exclusionary definition of *Section 2.1*. Para.4. Whereas an infinitesimal *ma* appears in the differentiation *operation* applied to abstracted singlet measurement points on some real background metric, the doublet position basis state of an interval reduces under actual measurement not to real metrical zero but, instead, to a complex *zero-point* vector in the underlying state space. When realised this limit is what we call a particle doublet, or a fermion-fermion scattering interaction. The theoretical function of a "particle" is to be one of a pair of labels on this limit in which transformations of unit interval cease to be real Lorentzian, where real (+*t*) and imaginary (-*t*) representations mutually cancel not to $t = 0$ (an absolute zero), but rather to a limit which is a zero-point condition inside a structure of plural zero points. Or, the underlying complex basis state is the *reason* why a real zero of interval (the null signal line) is not a Newtonian absolute zero of scale but a Lorentzian relativistic limit.

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25 We do not need at this stage to be able to say what the 'direction' is, only that an act of measurement always reveals it as having one.

2) We thus seek to associate rest mass with the existence of a directed vacuum potential which is a universal ultraviolet limit on improper translational transformations, and a functional real "particle radius" is thus to be a common limit on improper real observer transformations of any of *N* radii each properly of unit scale (such that the surfaces of *N* 'unit spheres' lying at unit radius from each of *N* graph vertices can be seen as the loci of *N* interpermeable "particles" of the underlying mass space). One can say that the essential complex polycentricity of a PM space structure containing both improper and proper representations is what both produces the antiparallellism of real (+*t* particle) and imaginary (-*t* antiparticle) phases - giving the null self-orthogonality of lightlike null lines in Minkowski spacetime confined on a proper bosonic phase - and simultaneously prevents their self-annihilation from being complete - giving the improper fermionic phase - or that there is a nonlocal defect in the symmetry of the Feynman spacetime representation of QM, a vacuum potential equivalent to a spacelike shift at every vertex, which is a zero-point fulcrum for the conjugacy of +*t* and -*t* which we associate in the theory with the emergent relativistic and quantum constants *c* and *h.*

3.) A Newtonian analogue of the articulation of the nonlocal PM graph would be the turning moment on a rigid[26] linear rod subject to asymmetrical forces. A force couple, say the torque on a compass needle in a magnetic field, is a special case. But then imagine that there is no fixed axis of rotation; further imagine that the force vectors at either end are arbitrary. How does an 'observer' on one end of such a rod find the true resultant of all torques of all forces acting on *both* ends? Without knowing the moment of inertia she cannot, even assuming that the rod's geometrical midpoint and centre of mass coincide perfectly. Fortunately Newton's absolute coordinate background supplies the axis of rotation and angular acceleration and with it all the quantities of force, mass and inertia become interdependently calculable. Everybody knows where they are. But if the coordinates vanish, what then? And what happens to inertia when different observers specify different relativistic forces? The modern field-theoretic idea is that the field carries the momentum and balances accounts for all different observers, which amounts to a conspiracy which distorts the coordinates so that any 'freely moving' (i.e., gravitationally constrained) mass always *appears* to be absolutely at rest. Indeed the concept of 'force' all but disappears in such a view. But inertia is not accounted for in a satisfactory physical way in this relativistic field theory of gravity.

4.) The PM system eschews the background-dependency of both the local field and the absolute coordinate background, replacing these ideas with a dynamical triangulation analogous to a linkage of nonlocal rods, where the transmitted forces are analogous to classical mechanical contact forces.

26 A PM element does satisfy the definition of a classically rigid object, in the nonlocal limit of all local transformations, as will be shown later.

Instead of a freely spinning needle, imagine a rod joined at its ends by free gymbals into a system of similar rods. As the system changes configuration and the rod rotates, imaginary 'observers' at either end can arrange to agree that their equal and opposite angular velocities cancel at the geometrical midpoint, and this will be valid for any arbitrary combination of forces; but this geometrical axis might itself be uniformly moving in another embedding frame defined by some surrounding sub-system of rods. There is no real 'still centre' of Newtonian forces on which to anchor an inertial frame. Evidently the closest we can get to an approximately fixed axis of rotation will be found in the frame of an 'observer' for whom the vector sum of *all* rod momenta in the *embedding* system is zero. Thus we can imagine that only in proportion as the sum is taken over an increasingly large number of interlinked rods will it become possible to identify an *imaginary region* which for practical purposes represents a single resultant for most observers inside the system; and only in the limit of an *infinitely* large system of rods of arbitrary lengths does the corresponding region approach an infinitesimal point - a unique axis - where the algebraic sum of moments can be considered to be practically zero, defining a point of functional 'rest' for *all* observers. So there *is* an 'inertial reference frame' in this picture, but it is not an absolute coordinate matrix and is rather a function of the linkage which exists between all accelerations. Therefore the forces that relate to it, whilst not fictitious, are always mediated throughout the entire system in a way that is not necessarily locally transparent. It is in fact not a static reference frame at all, but a dynamical reaction force which is a kind of feedback. In modern language this is a kind of *gauge symmetry.* Such a picture reconciles a classically reactive inertia with Newton's **Third Law** stating that *action and reaction must always be equal and opposite.* (Given the strict constraint of *Section 2.1.* para.4 that the PM geometry is to be a *complete* graph, the perfect rigidity of each rod would actually enforce $\Sigma \boldsymbol{F} = \Sigma\text{-}\boldsymbol{F}$ in the limit of zero forces for the whole system, in the sense that both kinetic energy and tensile stress and strain must be zero everywhere. This suggests that an invariance governing real observer-transformations of the linear element of unit scale in PM must have the character of an elastic modulus. See *Section 2.3.* for an interpretation of the Lorentz symmetry group.)

5.) This is a purely schematic illustration, of course, but we get the idea that the First Law has only approximate validity as a limit case in PM because the system of actions is self-limiting in some lower bound. This arises because 'force' cannot be separated from a scalar 'mass' on which it acts; mass, we say, *is* a force which acts onward (nonlocally, as a 'contact force') through the system. In contrast, in Newtonian physics we would be able to say that if we regard acceleration as zero by definition in the absence of force then we are left with a non-zero scalar quantity *m* which corresponds to a constant absolute mass. Special relativity deals with the total kinetic energy and the equivalent 'Newtonian' mass becomes an invariant of the energy-momentum transformations, or a 'rest energy'; however this is still a scalar. In PM 'mass' itself signifies an invariant minimum of force - associated with a pseudo-scalar unit vector which contains an 'absolute acceleration'. So as in GR we can say that 'inertial frames' are not truly equivalent. But in GR the non-flat affine connection makes acceleration a function of spacetime geometry which discriminates non-equivalent classes of inertial trajectories and removes the concept of force by, as it were, 'dispersing' its function into the continuum geometry. (Actually it is questionable whether this removes or objectifies the notion of force.) In PM on the other hand the linear projective geometry 'concentrates' force into a pseudo-absolute[27] and *intrinsic elementary property*. (This can be thought of roughly as a reincarnation of the Newtonian *vis insita* inside a relational theory.) The geometry dictates that not only is the product $m\boldsymbol{a}$ (trivially) a vector, the quantity m must itself be a directed vector. In other words, it becomes impossible to find a 'particle'-state where a scalar mass[28] can be separated from acceleration in measurement. This can be expressed equivalently as:

a) the condition of local 'rest' includes an acceleration
b) mass is equivalent to a spacetime displacement.

6.) The association of an *intrinsic* acceleration and an invariant length to the PM unit object with the condition

$$\frac{\overline{F}}{\overline{m}} = \overline{a}_{\lim} = \frac{\Delta_{unit}\overline{v}}{\Delta_{unit}t} = 1 \tag{6}$$

means that real time can be thought of as a result of some operation on unit time, just as real length occurs as a transform of unit scale. According to our definitions this operation will be a 'measurement' self-interaction that produces a *ratio* of finite real-number-valued multiples of unit time, so that an infinitesimal differentiation of a local quantity is a rational operation which does not imply *actual* differentiation of the nonlocal *basis state* of that quantity. Thus unit time rationally supplants the universal continuum role of the point-instant, retaining the essential functionality of the point to the extent that it is invariant under all linear transformations and remains *properly scale-free*. A primitive quantum of scale which is invariant under all scale transformations obviously has no unique determinate length at all! It brings with it only a primitive sense of a Leibnizian exclusion of two discriminable position states. But it is important to understand that

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27 To be clear, the 'absoluteness' relates to the unoccupied *unit vector* state dictated by our geometry; the 'acceleration' represents the mandated occupation of that state by the nonlocally-determined quantity.

28 We could describe the 'rest-energy' as a pseudo-scalar limit of a vector associated with a non-zero 'quantity of rest', which sounds a little strange but could be understood as meaning that the zero-point of displacement in PM (i.e., the limiting transform of unit scale) is a vacuum state containing a *negative scalar energy*. This idea attains a clearer form as we go on.

what is being proposed is no more empty of meaning than an empty geodesic in a spacetime theory. Indeed, the difference is that we are making this unit scale isomorphic in every case with a trajectory, so that *every* state becomes a filled state with a well-defined spectrum of metrical transformations.[29] It is possible to think of this procedure as assigning to each PM 'object' a complete 1-dimensional spacetime in which, to a notional observer inside it, a metre rod remains a metre rod whatever the inflation rate of the 'horizon scale'.

7.) This invariance therefore gives us the dimensionless identity between all lightlike zero-vectors. In other words, this is the straight 'geodesic' trajectory of a PM null signal line. But the same limit also occurs as an *improper* (local) limit of differentiability of unit length. In the proper, lightlike, limit the line expresses only the basic congruent transformation symmetry of our PM definition (i.e., it rotates identically onto itself; see *Section 2.1*. Para.4, n.9); this is a complex transformation and the set of all such lines does not admit of any real metrical transformations among itself. That is, all such states have interchangeable null identity. The improper, time- or space-like, limit is a limit for the isomorphic set of all such lines which *do* metrically transform. Away from this limit these metrical states are (by definition) not directly interchangeable because they project into one another only according to the Lorentz group of transformations; however, in the limit of non-differentiable unit scale - the "particle" scale - each line expresses its basic congruent transformation symmetry *interchangeably for all observers,* and so the only degree of freedom available for non-identical transformations - the registration of forces on the object by co-terminal objects - is rotational. In other words, rest-energy m itself corresponds to a universal lower bound on a transform of the PM unit length (which represents unit speed = c for all vectors), defining a characteristic scale at which this notional zero-point of momentum associated with 'rest' goes over to a torque producing rotational moments proportional to a range of *negative potential energies*.[30] We can express this by saying that the constraint which preserves unit length constant for all observers under rotation (which is in general the nonlocal system force responsible for mass) represents a *centripetal* acceleration of a pair of boundary points that delimit a unit length. And these two propositions combine to imply an invariant lower bound of angular momentum, which therefore appears as a 'fundamental' dynamical quantity. Away from this limit rotations of metrically transforming trajectories representing *positive kinetic energies* will generalise underneath the Lorentz group as relative curvatures.

29 Obviously we are not yet specifying the complete spectrum of 'particle' states - i.e. weak, colour and gravitational vector particles - that will do the filling. For these schematic discussions we are to think in terms of electrons and photons, except where stated, the gauge symmetry of QED being a useful heuristic paradigm.
30 Atomic electron energy levels are regarded as negative in somewhat the sense suggested.

8.) Leaving the latter generalisation to one side for the moment: According to Newton's law the inertia of a particle subject to a force ***ma*** is an opposite reaction force -***ma*** called into being by the acceleration. But relativistically the status of 'force' itself recedes in significance, so is inertia now no more than a book-keeping device? This would be useful, inasmuch as conventional relativistic field theory cannot account for inertia anyway. Conventionally therefore one might say that by including *imaginary* inertia we merely cancel an equally imaginary 'mass vector' and restore equilibrium at any point of measurement, restoring the idea of point particles in free space. But it is perhaps worth re-emphasising how and why in PM our foundational geometrical definitions disallow this: The mass vector is an *internal state* of a PM 'particle', a non-zero lower bound to the possible scale-transformation of the PM unit vector, not an addition to some recoverable scalar mass. To physically cancel the mass vector would be to cancel the volume-element of space and our isomorphic 'particle' with it, returning us to the unintelligible 'equilibrium' of a scalar mass-point in one of a degenerate infinitude of empty point states, which is precisely the case which we have striven to exclude in our definitions. A null resultant under this constraint of spatial non-degeneracy is very different from saying that the two antiparallel vectors are not physical states: In the latter case the residual scalar mass-point tells you nothing about force potentials, which remain freely specifiable in some arbitrary theory; whereas in the former case, in which a spacetime interval is included in the first order, the automatic implication is that potentials associated with a force proportional to mass vanish internally, as will be brought out presently.[31]

9.) So it is of the essence of PM that we cannot arrange for mass and inertia to simply cancel one another away (except for the class of proper lightlike null trajectories). And there are sound reasons for wanting to give a physical account of inertia. Assuming that the mass vector ***m*** is real we are forced to conclude that the inertial reaction term in an equilibrium equation of state for *a point in Newtonian space* corresponds to an actual force directed on *a line in PM space.* Newtonian inertia is then not due to a scalar mass coupling to any generalised coordinates but is a dynamical reaction force in a system of many linear 'mechanical contact' forces, in effect just the inverse of the set of actions which *generates* a vectorial mass, and the inverse, therefore, of that mass itself. This means that there is **neither a privileged class of relativistic inertial trajectories (geodesics) *nor* a privileged class of Galilean inertial frames (uniformly translating observers).**

10.) This is a subtle but important conclusion that takes us back to the issue raised in *Section 1*: In a Newtonian space-and-time theory, or a Minkowski spacetime theory, which both preserve *absolute particle-acceleration* against *extrinsic* coordinates, an *absence* of acceleration classifies *some*

31 The fact that they do not vanish *ex*ternally, i.e. that mass is a nonlocal property of closed *loops* of PM dyads, will be shown to be deeply connected with geometrical-topological phase effects of the Aharonov-Bohm type.

trajectories as inertial. In PM, on the other hand, the *universality* of absolute acceleration against *intrinsic* coordinates ensures that *all free trajectories* are inertial. This arises because momenta only change at vertices in PM, and all changes of momentum are rotations of unit vector through some phase angle, so that every origin embodies some new function of velocity squared. In a spacetime theory any arbitrary point of the continuum is a valid origin from which to measure, and it is always possible to 'find' some frame in which the instantaneous acceleration of a particle may be relativised away (if it has mass), its momentum notionally 'dumped' into the field, such that it becomes a pure timelike trajectory with a space displacement and momentum of zero. But in PM only another vertex is a valid origin from which to measure, and in the absence of instantaneous time derivatives every such vertical 'observer' has to associate some non-zero multiple of unit angular momentum with every other vertically-bounded trajectory.[32] It thus becomes impossible, as a result of the basic PM geometry, to support a clear distinction between 'non-inertial' unit vector accelerations due to an applied force and intrinsic or 'inertial' accelerations where there is *no* applied force. Given that the PM geometry is to determine for us the structure and dynamics of space and time displacements, we can see that this implies an extension or generalisation of the **equivalence principle** (EP) from a scalar mass to a vectorial mass-energy that includes some multiple of unit time squared. And from this we can expect to make a connection through special relativity (*Section 2.3*) to general relativity.

11.) It is interesting meanwhile to compare this proposal with the Mach-Weber theory of inertia for a universe of particles interconnected by nonlocal far-actions. Given the postulate that *the sum of all forces on a particle is zero in all coordinate frames*, Machian inertia arises from Weber's force law

$$F_{1,2} = -H_g \frac{m_1 m_2}{r^2}\left[1 - \frac{6}{c^2}\left(\frac{\dot{r}^2}{2} - r\ddot{r}\right)\right] \tag{7}$$

which modifies Newtonian gravitation by terms proportional to the relative velocity and acceleration. Here the inertia is a dynamical reaction force. Analyses by Assis (1993) and Assis & Graneau (1994,1995) show that the long-range $1/r$ force term proportional to the acceleration (the third term in eq.7) implements Mach's principle by effectively dividing the cosmic mass distribution into isotropic and anisotropic components. The long-range $1/r$ force locally will be

32 Another way of saying this is that no PM trajectory is open to infinity, which is analogous to excluding perfectly monochromatic wavetrains that have been propagating for an infinite time. In a universe that could contain such a wavetrain an infinite number of observers could always be found who would agree on its exact wavelength and momentum. In a PM universe of bounded states all waves must be polychromatic waves of mixed relativistic momenta, where an indeterminacy due to a superposition of wavelengths and amplitudes reflects the finite plurality of different observer 'frames'.

dominated by the isotropic gravity of the 'fixed galaxies', generating inertia as a dynamical reaction against the $1/r^2$ Newtonian accelerations produced by anisotropic nearby masses. An especially interesting result of the Weber force law (in the context of PM) is that in general the effective inertial mass of a body need not be isotropic, and will depend on the potential where the body is located.

12.) Weber's postulate means in a sense that Newton's First Law for a particle in static or inertial equilibrium is extended to particles with all relative velocities and accelerations, or in other words it applies to generalised *trajectories*. But of course this particle is a Newtonian mass point in free space; the inertia is then an abstract vector opposite to the particle acceleration, due to the $1/r$ attraction of the distant universe acting always on the point of the trajectory to maintain that point in dynamical equilibrium. Equilibrium then defines 'stasis' for the purposes of the First Law and we can say that gravitating frames are all inertial frames in the sense that their trajectories are conservative minima and define operationally the local inertial geodesic structure of spacetime. We have deduced for the nonlocal linear objects in PM that the condition

$$\overline{F} = m\left(\frac{d^2x}{dt^2}\right) = ma \neq 0 \tag{8}$$

has essentially the same meaning in relation to trajectories, but the distinction needs to be emphasised again that in PM the dynamical inertial term $-m\boldsymbol{a}$ has an actual representation in the local space structure. It is the inverse, or congruent geometrical transform, of an actual linear object (bounded by points of measurement) on which the limit of the time derivative of velocity does not go to $dt = 0$. There is thus no actual quantity of instantaneous acceleration, and instead of saying (as one may in a Mach-Weber model) that an inertial force produces an *effective* anisotropy in a scalar particle mass, one has to say in PM that mass is *intrinsically* a directed force, not a scalar, which has rather different implications.

13.) According to Weber's postulate inertial force is the dynamical reaction force due to the 'fixed galaxies' that restores the vector sum of forces on a particle with gravitational mass m_g to zero for all observers. We accept the spirit of this principle, so that

$$-\overline{F} = m_i\overline{a} = -m_g\overline{a} \tag{9}$$

Conventionally the sign is the property of the force vector and therefore vanishes when acceleration goes to zero, to leave just the identity

$$m_i \equiv m_g \tag{10}$$

which states the equivalence principle, a simultaneous identity of two indiscernible scalars which is an *unnatural relation* in Newtonian physics and an unrealised *identity in principle* in GR. But in our theory there is no longer an unnatural relation nor an identity in principle, but instead a conditional natural relation,

$$\vec{m}_i - \overleftarrow{m}_g = 0 \tag{11}$$

because m_i and m_g are force *operators* which ensure that the total mass-energy on any trajectory is always the *zero sum* of two antiparallel vector operations. These 'measurement' operations are conducted by the system of nature on itself not 'at a point' in a continuous coordinate space, but over a non-degenerate volume element of PM space; and the two vectors behave like the equal and opposite time-reversed forms of a discrete action. Or +*t* and -*t* are interchangeable by a congruent transformation of the line into itself. Thus

$$-\Delta t_{m_i} + \Delta t_{m_g} = 0 \tag{12}$$

and this is why the natural relation is conditional: The identity of m_i and m_g occurs as a null identity, which identifies it as a *proper* characteristic of the special class of lightlike zero vectors but not an *improper* characteristic of the classes of positive timelike or spacelike vectors. In other words, if we 'view' the graph of the universe from any given point of measurement (any vertex) we see that a symmetry preserved on lightlike (radial) directions is broken in their transformation to (transverse) displacement 4-vectors, with the emergence of positive-real time and mass. As we will show later this turns out to be the same as saying that mass is an emergent dynamical property of *plural* PM systems - minimally, triadic systems - and in this way the fact that non-vanishing positive mass-energy selects out transverse trajectories is intimately connected with the *many-centred* structure of gravitation in PM space. (It is as well to emphasise that the null mass vector *must not be confused with annulment of the scalar mass that couples to the universal gravitational field in GR*. We say only that a default state of equilibrium for a system conventionally regarded as 'two particles' is that *m* vanishes on the lightlike path between them. This is *not* the same as saying that this system feels no gravity or inertia, because the conditions of 'feeling' gravity and inertia are precisely those which destroy the default state of equilibrium.[33] In this sense structure controls mass, rather than *vice versa*. This relativistic space structure is developed in *Sections 2.3, 2.5 & 2.6*, showing that gravitational 'attraction' arises in PM as one pole of an emergent *dipole* due to a mass-field having these null longitudinal components.)

14.) Meanwhile let us further investigate the properties of the unit vector geometry, separating clearly the radiation field and matter field representations from one another (remembering that these remain representations of the same underlying supersymmetric object[34]). From bosonic trajectories as the definition of geodesic straightness we will go on to consider the case of curved fermionic trajectories in slowly-varying force potentials. Now we have seen that the condition of 'rest' includes an acceleration and that mass is equivalent to a spacetime displacement. These are natural conclusions when rest is a condition that has to be defined not for a 'mass-point' but minimally for a *pair* of co-dependent position states.[35] Note that because this limiting vectorial mass is to be a bound property of the related pair it may therefore be considered to *include* a notional 'mass' due to a photon exchanged by two electrons (just as the masses of atoms, nuclei, nucleons, mesons etc., often treated as 'elementary particles', are actually the vector sums of several component momenta). Emission at *A* and absorption at *B* shifts the balance of potentials in the bound system *AB*, but re-emission at *B* and re-absorption at *A* would shift it back with no overall change in the mass of *AB* - which paves the way for thinking in terms of the total relativistic mass-energy of the system.[36] From this point of view a formal photon 'rest mass' of zero reflects the fact that rest is not a condition available to a photon, in that it is a cursor that cannot be freed from confinement.[37]

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33 One might object, therefore, that this equilibrium is an ideal abstraction with no physical significance. The fact that nature is a plural system means that in general the condition for existence of the dyad is destruction of its equilibrium, of course. However it turns out, as we will see later, that extreme thermal isolation can approximate some properties of this abstract equilibrium.

34 At this stage we use the term 'supersymmetry' very loosely to express the idea that the 'mass particle' and the 'radiation particle' are just transforms of the same basis state. A defence of this proposition in terms of an interpretation of spin statistics will be needed later.

35 As indicated in para.13 we will be saying that the elementary unit of PM position space is a triangle of units scale, involving therefore a triad of points of measurement. In general, of course, one can infer that the limit of determinate position for any point of measurement involves a calculation over *all* the states of the network. See *Section 2.2* para.3.

36 Consider the emission of the photon from an atomic electron *A* in quantum state ψ which accommodates Newton's Third Law by incurring a proportional change in the kinetic energy of the emitting atom. This recoil represents energy 'stolen' from the photon. This limits the ability of another atom *B* in state ψ_1 to absorb the photon, as it has a longer wavelength than it 'should' have. From the point of view of electrodynamics the photon *is* absorbable, however, and materials are thus not perfectly transparent to their own radiation, 'because' the atomic energy levels are not perfectly sharp (recoil-cancelling thermal motions also play a part). From the point of view of confinement however this spread of atomic 'energy levels' is not an absolute 'given' property of separate atoms but is rather a graph of the small incremental variations in the total mass-energy of the system, including the photon. As it is not possible to reduce away this vector to an isolated scalar mass-point 'at rest' it is only possible to understand the absence of a photon as a time function of the total mass-energy, i.e. an 'exchange of virtual photons' becomes a periodic fluctuation in an amplitude associated with the whole system. The see-saw of electron energy levels accompanying exchange of a 'real' photon represents an incremental rotation of the whole supersymmetric oscillator. (See discussion of 'Einstein's box' later.)

37 There is an analogy with a quark-antiquark pair bound with a gluon string. The components of this object are not realisable due to confinement in QCD, and one can liken the photon zero-vector to a 'line of force' or an infinitely thin flux tube on which the force is independent of distance even though a statistical density of radial flux tubes per unit

Confinement is the paradoxical price of its individuality. The definition of freedom for a photon - a state problematically available in the conventional quantum field formalism, but evidently not in PM - is that it become a ghost, unobservable and virtual.[38] This suggests that a total electromagnetic mass-energy of 'two electrons' due to their energy of interaction exceeds the sum of the their 'bare' self-energies by an amount which is proportional to a virtual photon momentum density.

15.) In PM therefore a massless vector particle is only an abstraction from an energetic trajectory, a momentum; but there is a sense in which one could say that the First Law finds its proper meaning in PM for the case of massless particles that are always confined. Indeed the First Law is reduced to a truism for a photon in a PM space of irreducible lines, if those lines can be characterised as inertial trajectories. In a relativistic spacetime a 'free photon' follows the lightlike geodesics of a flat affine connection, or in gravity follows those of a non-flat affine connection, which in either case is just to specify the constraint. However there is no affine operator in PM space, no continuous geometry to constrain a particle to trace a trajectory. Instead we have a primitive linear object which *is* the trajectory; we make it impossible that a photon should be anywhere other than on this trajectory just because there is nowhere else for it to be. A photon's trajectory becomes a local operation, performed with a mathematical 'cursor' called a photon, on an underlying nonlocal object which has something of the character of a classically-rigid string.[39] So because photon momenta are confined to the minimal 1-surfaces of strings that *define* 'straightness' (by the definition of *Section 2.1.* para.4) we say that they must correspond to least-action paths of free inertial particles. Their trajectories only change momentum discontinuously at vertical intersections - i.e., *they only 'interact with charges'* - which suffices to represent the radiation part of the 'field'

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spherical area falls off as r^2. A non-zero photon rest mass is of course inadmissible in a relativistic gauge-invariant theory of particles for what are regarded as fundamental symmetry reasons; but technically it is excluded for reasons that relate to the absorption of infinitely divergent terms into electron mass/charge renormalisation in QED. The origin of this divergence can be traced to the separation of the particle and the radiation fields. PM on the other hand would be radically supersymmetric in a way that admits no such separation, insofar as the number of degrees of freedom for the 'field' in any interaction is limited to the number of doubly-connected position states or 'fermion pairs'. In other words the 'field' *is identified with* the sum of all such pairs, analogously to the intent of the old Feynman-Wheeler action-at-a-distance 'absorber theory' formalism. The crucial point is that this 'field' now, in a PM interpretation, would not only specify all electrodynamical degrees of freedom but all *spacetime* degrees of freedom as well; so instead of an integration operation over all possible paths generating divergences at arbitrary energies in the limit of infinitesimal loops, a redefinition of 'all possible paths' to a finite network of lines in PM implies that the QED integral (which is a scale-free operation concerned only with the *phase* of the amplitude, *not* its absolute magnitude) should be taken over what can be set as a finite number of discrete paths 'in' the whole of space. This interprets the question in *Footnote 4* as to how an intelligible causal structure can arise if degrees of freedom $x, y \ldots n$ are not available simultaneously to each 'particle' and to all particles equivalently: It is in fact only by removing the continuous coordinates and summing over paths nonlocally that a causal structure simultaneously exploring all degrees of freedom can arise at all.

38 Calculable effects due to virtual particles will have to be reproduced in PM if it is to work, as already mentioned. However, not all such effects are desirable. For example, the conventional energy of the vacuum due to virtual particles is supposedly infinite. In a 'flat' cosmos this leads to what has often been described as the largest single mismatch between theory and observation in the history of science.

39 Although in GR there is not overtly any 'underlying nonlocal object' it has been argued by some that there should be. For example, our elementary basis state of 'unit scale' would have an analogous function to that of the 'djinn' in Laurent Nottale's scale-relativity formalism.

as a network of massless null lines and underlies the *linearity* of the boson statistics.

16.) Consider now the fermion part of this supersymmetric field of charge-confined photon strings - the pairs of boundary conditions on every PM unit vector. These strings are objects that *may only be* 'picked up' by their fermionic ends, and indeed *always are* - that is, there are no 'free' ends in the network according to our founding definitions (see *Section 2.1*, Paras.4 & 8). A space of these objects contains no true Galilean inertial frames, such as might be associated with two free Newtonian particles each subject to a zero resultant of forces. There is actually no need for inertial frames in Newtonian mechanics either, of course, because Newton's absolute space supplies a privileged global frame of inertial coordinates available for all motions. But inertial frames become central in SR as the privileged class of reference structures in place of absolute space. Then in GR the function of reference structure is transferred from inertial frames to the affine geodesic. In PM the inertial privilege goes not to a generalised geodesic structure but to any trajectory, as a primitive property, and the 'reference structure' is the dynamical nexus of all trajectories. In the case of PM we find that instead of absolute inertial coordinates everywhere in infinite space we have a pseudo-absolute 'intrinsic' quantity which is a vectorial analogue of Newtonian scalar mass, an irreducible force quantity which appears with the dimensions of an angular momentum. Each element of unit scale is thus analogous to an individual inertial compass which, rather than aligning to a global 'north', *sets* local 'north'; but this 'setting' is not arbitrary despite the absence of the global field because it is collaborative through the primitive mechanism of 'contact forces' on our linkage of pseudo-absolute unit vectors. This is what supplies the key in principle to the understanding of inertia in a relational theory: The traditional distinction between an 'absolute' scalar property of a particle (Newtonian inertial mass) and a property of pure relations among an ensemble of particles (Machian inertial mass) relies on a distinction between fermionic and bosonic fields which disappears in our radically supersymmetric picture of extended vectorial strings, which are at once both (fermionic) 'objects' and their own system of (bosonic) interrelations.[40]

17.) Now the 'straight' line of Newtonian inertial motion has two new projections inside PM which

40 It is not possible to define an 'intrinsic spin' for our linear object so as to preserve a scalar rotational equilibrium in the sense of the First Law; it is limited to either an exchange of some labels attached to pairs of terminal states or to a spin coaxial with its 'trajectory'. The first type of intrinsic spin has an application in principle to the nonlocal context-dependent spins of 'electron pairs'; the coaxial degree of rotational freedom describes photon polarisation. In PM a rotational exchange of two terminal electron spins, or one half of a coaxial photon spin, turn out to be equivalent descriptions. We can make our abstract model a little more physics-like by recalling that in QM the quantisation of the EM field allows the photon wave function to fill all of space in all of its modes, where 'all of space' can be the confines of a perfectly reflecting box. PM space offers a perfectly-reflecting 1-dimensional box for the photon wave function describing the 'exchange' of a 'cursor-particle' between each two points of measurement (or vertical changes of momentum). These latter are the twin boundary conditions of the unit scale vector that take the role of a 'pair of electrons' (or, in the first available mode, a transient electron-positron pair). The fermion spin associated with these termini is a more complicated function of emergent orientations and is discussed later.

illuminate the meaning of associating mass and acceleration with an element of directed unit scale. Firstly, since we are operating with a minimal definition of a line as the projective dual of a point we are safe to regard this as our 'gold standard' of straightness. A 'curvature' in the sense of an actual deformation of the *internal* symmetry of the line requires not only more information, but specifically an infinite number of additional real coordinates in order that by coupling to them in interaction (an observation of some kind) we might be able to discriminate the associated change in some ratio of quantities (whatever they might be) from a variation due to some other functions that we might imagine being attached to the line (i.e. time, velocity). In our construction a first-order deviation from straightness would in fact be the same as the interpolation of a *third gauge coordinate* and the production not of a curve in one line but of *two new straight ones*, or: A discontinuous acceleration occurs at the point of application of a force.[41] As we add more coordinates, each *vertex* of the emergent graph generates a new copy or 'avatar' of the line *renormalised*[42] for the purposes of Newton's First Law, in accordance with the Second Law. (Remember, we are unable to carry this process to the limit of a differentiable smooth curve because no single path segment satisfies the notions of instantaneous acceleration or instantaneous scalar rest. A smooth curve is a non-rectifiable improper transform of a properly straight 'object' defined by no more than two points of measurement. Conversely no sequence of >2 real measurements ever goes over into a smooth curve because integration always ceases in the limit of a series of vectorial 'rest masses'.) Each finite new trajectory is then available for analysis either in terms of its complementary lightlike, timelike or spacelike aspects.

18.) We already know that in general the PM unit vector cannot be considered to improperly transform so as to satisfy the First Law for a scalar inertial equilibrium. There is a definition of straightness that satisfies the First Law for the *proper lightlike* representation of any trajectory, but this definition must be transformed under a conservative law of all possible *curved* trajectories in PM for all possible observers, because of the inevitability of a relative rotational moment on what is in effect a classically rigid body (being the definition of straightness as properly considered in its own frame), as mentioned above. And we can now see that it is this very nonlocal rigidity of the underlying PM string - paradoxically - that allows the possibility of the *orderly* variations in length

41 From this we can deduce that the 'rigidity' in our construction is the precise inverse of 'measurement' and so characterises states of the system which the system itself never realises in its own self-measurement. This suggests that the elementary objects in our construction *contain in themselves* and *express in their relations* states of the system that are *virtual states* of the system. This promises to be a useful property of PM space because, if it turned out that these were the analogues of QM virtual states in a PM theory that was fully mathematically equivalent (a large 'if', admittedly), then the equivalent virtual quantities would not go to infinity. In other words the network is a simple graph and a complete graph with neither multiple edges *nor loops*.

42 We will show later that the line is rescaled at the vertex by a mechanism which acts to conserve, rather than pure real distance, a complex vector quantity associated with it. We propose to use this process to interpret a *general* vertical renormalisation of physical 'constants'.

scale and curvature which bring in the relativistic mass and acceleration associated with timelike displacements. The radiation part of the 'field' acts as a gauge for the mass-particle (fermion) part of the field. In other words, the lightlike vector is a portable constant of all pairs of 'charges exchanging photons', independently of changes of scale or motion, and a 'curvature' of the photon signal line is an imaginary curvature only, belonging not *properly* to the photon but only to its *im*proper representation by a mathematical *cursor*.[43] But the same is not true for the part of the 'field' that describes the entire supersymmetric linear object with its included mass - i.e. if we imagine a displacement of one 'end' of the string from the other 'end' in a slowly-varying force potential. This lengthening (or shortening) of the trajectory introduces a curvature precisely *because of* the underlying rigidity of the object.

19.) The reason for this is that *only* the assumption of underlying proper rigidity - which equates to the null lightlike line - will allow well-behaved transformations of the trajectory between different relativistic frames, inasmuch as an arbitrary proper 'flexibility' would be equivalent to admitting completely undefinable position bases for the emergent unit length, and by extension for the total energy. Or from an opposite point of view: The line metric consists wholly in the sum of the inter-transformations of unit scale among all observer-frames, and a limiting factor in the definition of 'rigidity' for the string will play the role of *c*, the speed of light, in the transformation equations. We notice, however, that this also implies an obverse limit in which the nonlocal rigidity of *each* string is representable as just the Fourier equivalent of *all* superposed local curvatures.[44] This interprets the paradox of an 'absolute' speed at the centre of a theory of 'relative' motion. It is an effect that could not exist for a space of classical particles, and one which can be seen as embodying the twinned spirits of Mach's Principle and Riemannian spacetime in the linear geometry of PM.

43 Evidently if we say that the progress of the cursor represents a time-dependent evolution of the state of the line, and if we say that the cursor in this case is a photon, then we have to accept that we are asking the photon to 'react to a force' acting in the 'future' of the line about which it could have no information by means of a timelike signal. But then since the invariant length of a lightlike photon 4-vector is zero this is not so strange. Our construction is fundamentally nonlocal, but it is known experimentally that a theory which seeks to be dual with quantum mechanics cannot be a local theory. EPR correlations prove this. The Cramer 'transactional' model of the wave function using both the advanced ($-t$) and retarded ($+t$) potentials is arguably the most intelligible version of the present formalism of QM and eliminates positive-time chauvinism without violating the 'no-signalling' condition for Lorentzian measurements.

44 And this implies a democratic nonlocal system contribution to the common 'constant' index of string rigidity (a function of mass/ length^{-1} and string tension, in conventional terms) which we have deduced is associated with the quantity *c*. In other words, we expect that the *virtual* energy state on any *one* string, or equivalently the vacuum energy, will appear as in some sense a sum over all the *real* states of all *other* strings. (See *Notes 32* & *35*.)

2.3 consistency with relativistic mechanics

1.) How can a theory of nonlocal 'far action' forces possibly be dual with the pre-eminently local 'field contact' theory of relativistic mechanics? So far this possibility has been asserted but not demonstrated. Here we will try to do so. First of all, in general, how can we assert that PM requires breaking of Lorentz symmetry and at the same time has consistency with SR? This question should be seen in context with the fact that the existence of structure *per se* is a breaking of Lorentz symmetry. The group contains no preferential space direction, and indeed is invariant under CPT reversals, yet nature is full of 'preferred' directions in spacetime, due to mechanical forces, gravitation, intrinsic spin vectors and so on. The structure of a crystal, for example, represents breaking of rotation and velocity transformation symmetries for a particle moving inside it. Lorentz symmetry is strictly-speaking spontaneously broken for every realised (directed) trajectory. But this doesn't seem to be what is usually meant by a broken Lorentz symmetry. The usual tacit assumption is that if you could see the underlying pattern of *unrealised* directions centred on every infinitesimal point in space then it would in every case be perfectly Lorentz symmetric, no matter that few of these possible states are ever filled by observable particles.

2.) This is expressed by the idea that the patterns which break symmetry in matter do not reflect the existence of pattern in the vacuum; that is, the vacuum is a perfectly isotropic continuum, a blank unwritten sheet on which true Lorentz/CPT symmetry is preserved. Theorists have considered possible breaking of the vacuum symmetry and have looked for evidence of subtle anisotropies that might reveal fine-scale 'graining' of the spacetime background underlying particle trajectories, so far without success. But clearly in PM the idea of a background graining has no meaning, because there is no homogeneous continuum of vacuum quantities to be specified. Rather, every trajectory *is* a spontaneous breaking of the (linear) vacuum symmetry and there are no empty vacuum states.

3.) The issue of how locality can coexist with a space structure of nonlocal forces is more subtle, but in essence it can be understood by reference to the problem of the 'ether'. As is well known, special relativity did *not* eliminate the possibility of an ether but made the hypothesis redundant by ensuring that the result of any experiment designed to reveal a motion relative to the ether would be the same as if the ether *a)* did not exist or *b)* conspired always to share the same kinematic frame of reference as the experiment. It should be noted that general relativity lays a similar condition on an experiment designed to reveal a distinction between inertial acceleration and a gravitational field gradient. In GR the conspiracy mediated by the non-Euclidean spacetime geometry is closely analogous to the conspiracy which SR requires to be mediated by the ether. In neither case can any direct measurement reveal the distortion. In the case of the ether, relativity dismisses as superfluous the presence of a 'metric' that transforms unmeasurably underneath the Lorentz equations; in the case of spacetime curvature, relativity welcomes the unmeasurable metric tensor as a boon which 'explains away' the *force* of gravity. The essence of both is that they elude calibration. In GR the force of gravity is superfluous, only the action of gravity remains, and the motivation of this programme is that the action is supposed to be locally specified in a theory that explains inertia. But GR is not properly local, and does not explain inertia either.

4.) So it is with all this in mind that one should look at the question of SR's supposed incompatibility with a nonlocal physics of far actions. In fact a great deal of physics appears to be nonlocal. The spin parameters that describe atomic and molecular structure in accordance with Pauli exclusion govern nonlocally; EPR correlations are ubiquitous in spin interactions and are routinely demonstrated over large distance scales. The Aharonov-Bohm effect and related quantum topological phase effects are well-studied examples of nonlocal far action. What does this mean? If we consider the causal structure of SR in terms of the light cone we can see that *precisely because c is a finite constant* it is possible to define the section $x = \pm ct$ of the lightlike hypersurface in Fig.5 in terms either of the past-future timelike zones **or** of the zone of elsewhere, which vanishes as c goes to infinity. From a certain point of view the local causal structure of relativity only exists because it is the obverse of an embedding nonlocal structure. The function of c is to enforce the *equivalence of causal physics* for all different observers by enforcing the *non-equivalence of observer locations* according as they have the possibility of being spacelike separated. It is in one sense 'obvious' that the relativistic *simultaneity* of two points joined by a spacelike vector *forbids* by definition the exchanging of timelike or lightlike information between them. Less obvious, perhaps, is the fact that rendering all intervals timelike or lightlike by letting c tend to infinity would mean that *no* two points could be improperly simultaneous; *every* measurement would be completely determined by its local connections to every other state as far as past- and future-timelike infinity. So the fact that origins of timelike intervals can be spacelike-separated objectifies the notion of 'chance' and is thereby fundamental to the causal structure of quantum theory. In terms of quantum electrodynamics, locality of the electromagnetic gauge field *is expressed in* and *is expressed by* the fact that Pauli exclusion *nonlocally* enforces the non-equivalence of pairs of electrons.

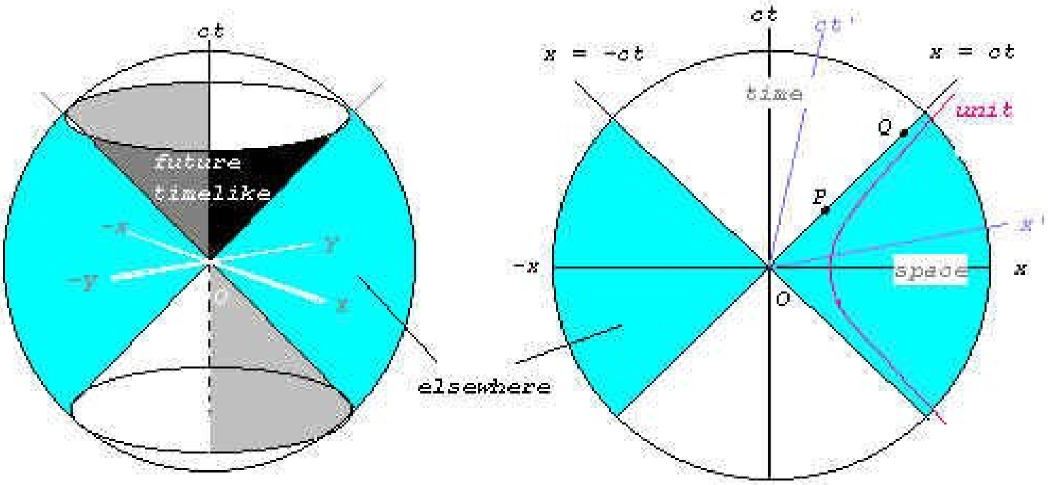


Fig.5. Causal structure of the light cone and Minkowski geometry, showing one branch (red) of the spacelike 'internal hyperbola' of unit distance

5.) From the point of view of PM this Leibnizian non-equivalence of displaced points of measurement enforced by *c* can be seen as the true meaning of Pauli's exclusion principle and is the essence of the non-degeneracy of position states in PM. In other words, nonlocal physical structure is not by any means ruled out by SR: nonlocal connections coexist peaceably with SR because of the 'no signalling condition' which rules out the possibility that any causal connection should *appear* instantaneous to observers attached to different fermions moving at <*c*. The cost of this is to create a class of bosons invariantly 'travelling at *c*' to which no (real) observers may be attached. This class supplants the function of the undetectable ether, insofar as attaching an observer to a boson would make instantaneous contiguity an observable, just as attaching an observer to the ether would make absolute motion observable.

6.) Far from it being the case that a Newtonian nonlocal structure was demolished by SR, it is obvious that the concept of nonlocality would have meant nothing to Newton because it only appears with the restriction imposed by relativity - the finite constant speed of light. SR brings into existence a class of relations which *appear from the point of view of local observers* not to take part in the causal structure of the world as defined by positive-going timelike vectors. But not only does SR *not* require that this appearance is actuality, SR itself provides a frame (*c*) in which all spacelike and timelike intervals rotate into one another identically. And in doing so it provides a new degree of freedom for the causal structure which Newton could not have dreamed of (even though it is implicit in the symmetry of his own equations of motion) - and that is, time-reversal symmetry.

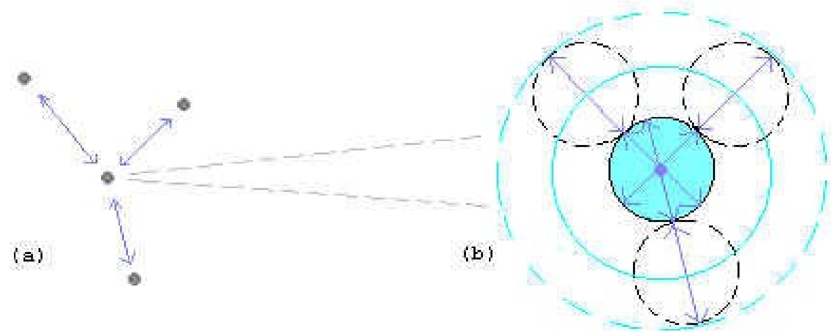
7.) In Fig.5 the coordinate system (*x,ct*) is a Minkowski-orthogonal pair of axes bracketing timelike and spacelike regions which are causally distinct for an observer at *O*, the distinction in terms of *c* being the signal velocity required for a causal connection: <*c* on the timelike side, >*c* on the spacelike, the latter implying *reversal* of $+t$ to $-t$ for a class of observers. In general spacelike causal connections with $-t$ are 'forbidden', in the sense that all observers must be able to agree on the same causal order - this so as to preserve the uniformity of the 'laws of physics' in all frames, which after all is what relativity is supposed to be for. But this says nothing about the ontology, only that the resultant of some *complex* combination of time operations of *opposite* sign - a superposition of advanced *and* retarded solutions of the quantum wave function - will have a positive real outcome in the 'forward' time direction. Quantum mechanics in fact goes so far as to *require* this complex time representation, in the form of a procedure for reduction of the state vector which is technically equivalent to mixing ($>c, -t$) and ($<c, +t$) wave trajectories.

8.) Para.7 implies that if QM is to be relativistically invariant then SR itself must be able to represent such complex entities, as of course it does. In Fig.5 another inertial frame moving with positive relative velocity in the x direction has Minkowski-orthogonal axes (x', ct') which are approaching the lightlike null line $x = ct$. In this and in *any* other case (x'', ct'') the space and time axes remain perpendicular. For the case of an interval PQ lying *on* the lightlike line $x = ct$ (common to all frames) it is said that the interval PQ is *perpendicular to itself*, because space and time axes must coincide, and its invariant 4-dimensional length s^2 vanishes to zero. This result can also be expressed by saying that the calibration hyperbola of *unit distance* from O in Fig. 5 intersects $x = ct$ at infinity. The photon zero-vector can therefore be seen as the null superposition of a $<c$ timelike and a $>c$ spacelike vector, or of *positive- and negative-timelike* velocities. This represents the fact that in QED a photon is its own antiparticle.

9.) Rather than eradicating 'instantaneous' far actions, relativity provides a *rigorous physical interpretation* of how 'instantaneity' works, in place of Newton's naive extrapolation from sense experience. It now has a more limited meaning, but a precise meaning. Instead of being realisable in principle for all classes of relations in a system it is now realised in practise for one very well-defined class of relations. What relativity does is to reconcile 'action-at-a-distance' forces with 'contact' forces by replacing both with the concept of action-at-*no*-distance, and the proper and improper views of the network of photon signal lines realise the projective transformation of point and line in the PM geometry. Our understanding of relativity in PM is that the vanishing of the interval on null geodesics represents the action-at-no-distance *between* trajectories, or the 'contact force' between contiguous ends of PM unit vectors. The unit vector enters as the value-free *basis state* of all relative scales, not as an infinitesimal differential, and it is the self-interaction of the system of these unit objects which occurs (at vertices) at 'no distance'.

10.) Very schematically, the *point-representation* of PM supersymmetry as 'seen' by a system of N

photons can be thought of as a 'lattice' of $N^{½}$ particles with a zero packing distance, each of which has 'internal' negative dimensions projecting along imaginary extensions of each of N photon spin axes. (See Fig.6) Each internal dimension represents two degrees of complex freedom in the form of a pair of antiparallel state vectors. (These will be pairs of basis states for electron spin in $2N^{½}$ 'directions'.) The complementary *line-representation* projects these internal dimensions on lightlike 4-vectors whose antiparallel components become advanced and retarded actions in negative- and positive-going time directions. In the line-representation, then, the nulls 'inflate' to lines of any and all possible real scales on positive PM dimensions (spacetime intervals become real) whilst reciprocally the point retreats to the status of a connection occurring at zero-distance at vertices between *bosonic lines*. In the point-representation, on the other hand, it is the line that shrinks to the status of an interaction at zero-distance, between *fermionic points*, whilst the points 'inflate' in internal negative PM dimensions (spacetime intervals become imaginary).



schematic idea of projection between line and point representations of PM dimensionality. Positive-real lines (a) are projected 'internally' as negative-imaginary lines (b). Photon time collapses both to a null-complex representation which effectively attaches an array of pairs of antiparallel vectors to each point of measurement.

Fig.6

11.) In this way we get the idea that a 'supersymmetry' appears to *us* to be broken because our nature as *plural systems of vertices* embodies the PM exclusion principle and the SR locality condition. The 'microscopic reversibility' of processes in Newtonian and quantum systems appears to imply that the reversibility is itself a scale-dependent principle; but PM has a scale-free reversible symmetry isomorphic to its scale-free elementary object, and it is only in the *registration* or 'measurement' of states that a distinction between 'microscopic reversibility' and 'macroscopic irreversibility' emerges (the implication being that entropy has an altered status in PM; see *Section 2.4.*). The necessary logical structure of relational 'measurement' mirrors a necessary physical structure which is 'interaction', a structure which is the essence of the many-centred 'observing' system(s) we are. The underlying complex PM geometry reveals that 'radiation' and 'particle' components of the field are projective representations of the same set of basis states, projections that simultaneously enfold and expel one another. We can describe this point of view as a relativistic dynamical supersymmetry.

12.) By rotating between these reciprocal perspectives we are able to see (still in a schematic way) that the plural light-cone structure is an interlacing of local and nonlocal views whose self-consistency is somehow crucially dependent on the fact that the world is a many-centred system in the sense of PM's 'exclusion' of degenerate states. We can express this in terms of the light-cone by saying that it is always possible, by an appropriate choice of observer coordinates, to make two spacelike-separated world-points coincident in time **or** two timelike-separated world-points coincident in space, with $ct = 0$ **or** $x,y,z = 0$; but it is *not* generally possible to obtain $ct = 0$ **and** $x,y,z = 0$ between the same pair of points. This identity is only realisable for the set of all points in a lightlike relation to O (like P and Q in Fig.5). Otherwise, the two sets of hyperboloid sheets of unit distance and unit time for two origins O and O' in two real observer frames S and S' do not intersect simultaneously, expressing the fact that any real relation of O and O' has to be spacetime non-degenerate and that our world-representation is therefore irreducibly dual.

13.) An implication of this duality in PM is that Newton's Third Law cannot be strictly true for the point of application of a force. Taken over the *entire* interaction path the action and reaction will be antiparallel; but this means in the limit of arbitrary path length and complexity, in the frame where the vector sum of *all* momenta is zero. At the points of measurement there will generally not be found an equal and opposite reaction. Ideally the Third Law would be obeyed in the Newtonian inertia of a scalar particle and this idea is realised in a non-relativistic Machian theory where inertia is a dynamical reaction force due to a Weber-type potential. But in relativistic mechanics, as in PM, momentum is not rigorously conserved at the point of interaction.

14.) In SR we are forced to admit the non-conservation of instantaneous momentum in any given frame. Only between the start and end of an interaction is it possible to say that momentum is conserved for all observers, which involves a certain arbitrariness; and between initial and final states some pair of 'interacting particles' always violates, instant by instant, the law of conservation of momentum. Conservative order is restored only by invoking the concept of the field as carrier of momentum and energy, so that descriptions of the state of the field in all frames may be so constructed as to conserve the total momentum. Of course invoking the field brings with it other problems, particularly when it has to be quantised with the result that the vacuum energy goes to infinity. In PM the particle action or local group velocity of the near field is supplanted by the unit vector, and the operation equivalent to invoking the infinite degrees of freedom of the far field would be to invoke the superposition (in imaginary time) of *all frames*.

15.) The non-conservation of relativistic momentum in Para.12 is closely related to the supposed prohibition of action-at-a-distance forces by SR. This prohibition consists in the statement that it is not possible to identify a *unique* form of the interaction between two particles, meaning that action-reaction equilibrium cannot be given a clear meaning for all observers *at an instant*, therefore instantaneous far actions have no objective physical status. Insofar as SR is only able to retain the concept of a rigorously conservative 'force' by invoking the local field as book-keeper for the transition momentum during interactions, then it has indeed given up the prospect of maintaining the Third Law for measurable 'particles' except over infinitesimal distances. Yet the situation in PM, as we have seen, is that Newton's Third Law cannot be satisfied for an elementary object in the theory precisely *because of* an underlying structure of nonlocal 'contact forces' due to which there are *no* infinitesimal distances between points of measurement. There is no paradox because an underlying nonlocal structure that transforms isomorphically to the relativistic interval has the same status as a dynamical ether and may do so without violating definitions of observer time within an SR system. But is not such an underlying matrix merely as superfluous as the ether unless it introduces some superadded 'force' contribution to the system? And if it were to do so - say, by way of the geometry of this matrix - would not this force by definition be superadded to, and therefore outwith the control of, those general transformations governed by the Poincare symmetry of SR? This is indeed so, and the underlying structure then acquires the same status as the GR metric tensor.

16.) Newtonian gravity proves that a very effective theory (though, in this particular case, a slightly imperfect theory only accurate to about one part in 10^7) can be made by assuming a network of instantaneous far actions; it doesn't matter that differently moving observers inside the light cone cannot share a frame in which instantaneity is measurable. The very structure that excludes them from such a frame by denying them a common definition of simultaneity ensures that there *is* one, traced by the network of null photon lines. As mentioned above, relativity gives a physical meaning to instantaneity in the form of a scale-free function of a finite velocity, replacing the Newtonian conception of instantaneity whose physical meaning depended on the idea of an instantaneous time derivative of an infinite velocity. The latter can be thought of as the vector resultant of an infinite series of positive-going $+t$ trajectories; the former, as the vector resultant of a finite series of $+t$ trajectories and their time-reversed $-t$ conjugates. The Newtonian instantaneous state is the irrational end of a divergent series; the relativistic zero-vector is the rational end of a convergent series. The affinity between PM and SR is self-evident here. In PM the null resultant occurs for any finite number of whole iterations of the complex process +/-*t* and expresses an *intrinsic* limit due to a cancellation of intrinsic vectors, which can never be reduced away to an instantaneous acceleration for any series of addition of velocities

17.) Another point about transformations under the Lorentz group: It has already been pointed out that the Newtonian scalar quantity *m* occurs as a vector in PM so as to modify the force vector under acceleration. In other words, in Newtonian mechanics the time derivative of the momentum, or the force *F*, is in the direction of the acceleration; but this will not hold in PM because the quantity *m* is itself an indeterminate vector associated with a non-zero extension, *x*. The trajectory *x,* which has to be treated as an elementary whole because of its underlying nonlocal objectification, does not satisfy Newton's first law for a mass point in scalar equilibrium. The force in general cannot sum to zero and the resultant will be a torque. Momentum is therefore always the vector sum of a resultant in the *x* direction which is an invariant for all observers *and* an intrinsic pseudo-absolute acceleration which is an indeterminate component in the transverse *y* direction.

18.) This is consistent with the way Newtonian forces transform in special relativity, where the acceleration is not an invariant. If we take Newton's Second Law in the form

$$\bar{F} = \frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v}) \tag{13}$$

where

$$m = m_o(1 - v^2/c^2)^{-1/2} \tag{14}$$

is the mass of a particle instantaneously at rest in a frame with velocity *v*, then parallel and transverse forces $\boldsymbol{F}_{0x}$ and $\boldsymbol{F}_{0y}$, producing accelerations $\boldsymbol{a}_{0x}$ and $\boldsymbol{a}_{0y}$ as measured in the rest frame of the particle, do not both transform invariantly in a laboratory frame with relative velocity -*v*, where

$$\frac{F_y}{F_x} = \frac{1}{\gamma^2}\frac{a_y}{a_x} \tag{15}$$

so that the ratio of the components of the force is proportional to $1/(1- v^2 / c^2)$. Only for the special case $\gamma = 1$, in the 'instantaneous' rest frame of the particle, would the time derivative of the momentum always be parallel to the acceleration.

19.) In PM there is *no* true instantaneous frame, except the proper frame of a lightlike zero-vector, so the forces are in general never parallel. SR admits an instantaneous proper particle frame but forbids any improper observer to share it. That is to say, only by *being* that same particle would 'another' particle do so, which of course would mean that it was not an improper observer, by definition! Imposing the condition $v \leq c$ for any timelike fermion in SR is constructively to deny the improper observer the ability to *freely* transform her view of another arbitrary particle into an instantaneous identity (i.e. by bringing both into a lightlike relation), and by means of the term γ it quantifies the infinite energy cost of the task of attempting to recover such an identity. Thus we can see that a relativistic electrodynamics which has discrete quanta of charge automatically implies an exclusion principle for fermion position states. The underlying structural reason is illuminated from the perspective of PM.

20.) In SR one says that a particle with mass cannot be accelerated to a velocity c. In PM this becomes the fact that, as 'observed' from *any singular point of measurement* (or vertex), all vectors where $m_{\mathrm{i}} + m_{\mathrm{g}} \neq 0$ (i.e., vectors that acquire non-zero observed mass) are *transverse* vectors, neither of whose boundary conditions is the point of measurement concerned; i.e., they are not longitudinal vectors originating at the point of measurement. We can see that this distinction corresponds in our scheme to the distinction between fermion-dominated and boson-dominated representations of a PM supersymmetry 'broken', for any one vertical point of measurement, by the existence of a plurality of such vertices (para.11). And so transforming a massy *transverse* component into a mass*less radial* zero-vector component of the field is equivalent to *realising an identity* between points of measurement with different position states. Bringing points A and B into a common lightlike relation with C would violate PM's geometrical 'exclusion principle', which, by forbidding multiple edges in a simple and complete graph, ensures non-degeneracy of position states in n-dimensional PM space. The positional point-state at A is not an independent locale which may be vacated and then refilled by B, but rather it is a relative function of the system *including A and B* (it is in fact a 'half-position' state in the PM geometry), so the dynamical transformation in which we might seek to introduce A into the position state of B is the very action that ensures that the latter is no longer available.

21.) In this context we can see that the local limit c has the same exclusion function in SR, precluding the degenerate simultaneous identity of two position states in 4-space. And, most important, acts of actual 'observership' on some state, acts in which *ratios* of quantities appear and are registered, occur inside systems composed *both* of massless null radial *and* of massy non-null transverse components. In other words the included angle at the vertex of *two* components conjoint with a *third* component is integral to the physics of 'observation', and the inclusion of this angle is equivalent to 'breaking' (or rather, expressing) the PM geometrodynamical supersymmetry of boson and fermion. Moreover, the process is an irreducibly plural and *mutual* activity involving an exchange of roles from vertex to vertex to vertex which rotates the 'mass' around the triad as the scalar product of three different pairs of vectors. Thus the triad is the *minimal* symmetry group for the emergence of a nonlocally-distributed dynamical quantity called 'mass'.

22.) We can illustrate the natural relation in PM between a finite wave speed *c* and the exclusion of point measurements (or the *in*clusion of angle) in terms of the physics of simple strings. If we think of *c* as characterising the *proper* null lightlike 'rigidity' of the unit vector (see *Section 2.1.* paras.18 & 19) we can then also think of it as a dynamical constant of *improper* states of 'tension' or 'compression' (see *Section 2.2.* para.3). Where these force vectors are exactly antiparallel the equilibrium condition of 'rigidity' will be a constant whatever the improper lengths of the vectors. Thus for the lightlike case we have the cancellation $-\boldsymbol{m}_{\mathrm{i}} + \boldsymbol{m}_{\mathrm{g}} = \mathbf{0}$ (see *Section 2.2.* para.13) and the proper equilibrium will be a pseudo-scalar constant of any null, longitudinal, massless, radiation vector, which qualitatively describes *c*. The problem is that *c* is supposed to be preserved not only on the massless longitudinal zero-vector but also for all different frames in which it undergoes arbitrary improper *tensile deformations* as a massive transverse fermion vector - i.e, not only for *A* and *B*, but for *C, D, E . . . n* as well. If the relativistic transformations involving *n* different velocities are analogous to varying string tensions, then the wave speed becomes a variable too, so how is it possible to maintain that *c* is a relativistic constant for all observers?

23.) As a preamble to that question, consider that in a system of relative scales a spectrum of states of positive tension is constructively equivalent to a spectrum of states of compression, except that whereas a compression of two position states could go to a degenerate limit, it is easy to define a natural lower bound to tension. That is, an infinitely large compression implies the inevitability of collocation (singularity); an infinitely small tension does not, and preserves duality at the point where the string 'goes slack'. But even so we do not wish to realise this limit of slackness: Obviously a string with zero tension has a wave speed $c = 0$; but more fundamentally a string with an absolutely zero (or absolutely negative) proper tension is unintelligible insofar as it violates our founding definition of **straightness** (*Section 2.1.* para.4). We cannot admit this because arbitrary degrees of freedom would then have to be introduced to map an arbitrary number of indistinguishable (because unobservable) configurations of the string defined by (and *only* by) the same two points. This would import an infinite number of 'virtual' states, and thus would merely substitute a tensile degeneracy for a compressive degeneracy. Fortunately, the above argument reveals that 'tension' is just another way of defining 'straightness', and we are therefore able to stipulate that a minimum condition of *all* intervals is to be a state of positive proper tension, meaning that states of measured 'negative tension' will always be states of *relative* tension which (like curvature) occur as *improper* observer transformations. So a string's minimal proper tension acts as an invariant lower bound to a range of relativistic energies corresponding to improper string tensions. This irreducible proper tension, then, looks very much as if it might behave like a 'rest energy' associated with a non-zero minimum of a *range* of wave speeds.

24.) Now, from the point of view of a *single string* considered in isolation the relativistic mass-energy relations involving *c* are easy to understand in this way. The relation between mechanical wave speed and string tension is

$$c = \sqrt{\frac{T}{\mu}} \tag{16}$$

where T = tension and μ. = mass/length^{-1}, and we can see that if string tension is allowed to vary then in order that c be held constant mass/length^{-1} has to vary in direct proportion (Fig. 6). So it automatically follows that if the total mass is a constant while string length (a photon 'trajectory')

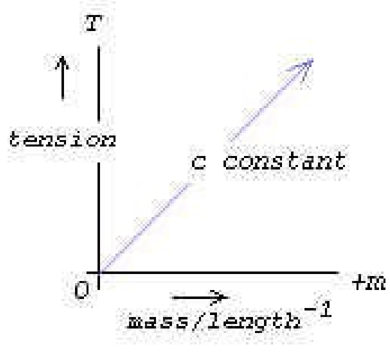


Fig. 6. Constant *c* and relationship between string tension and mass-per-unit-length.

increases or decreases proportionately to tension under 'force' transformations, then *c* in turn cannot be constant. But since we require *c* constant for all deformations of the string these changes in measured string scale ('tension' and relative 'compression') will be described by a relativistic dynamics of the system, to include Lorentz dilations and contractions, in which the total mass varies exactly like the total energy E, which we can show is the same as T.

25.) With mass/length^{-1} normalised to μ = unit mass = m for unit length, we have

$$m^{1/2}c = \sqrt{T_0} \tag{17}$$

or

$$T_0 = mc^2 = E_0 \tag{18}$$

where T_0 is just unit tension or the proper tension of an 'untransformed' string of PM unit length. So we have that unit tension equals unit energy, or T_0 is equivalent to the relativistic rest energy E_0. Now remembering that this unit 'rest' energy occurs as a stationary condition of the whole non-differentiable string, not as a property of a point-event, then we get the total scalar energy E of the string under relativistic displacement by the transformation

$$E = \frac{T}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19}$$

where v is the component of velocity parallel to the length of the string AB relative to a displaced point of measurement C (Fig.7). A positive energy E_0 occurs as an invariant limit corresponding to a minimal proper tension, and we conclude that states of *negative proper* tension would be negative mass-energy states, which will therefore correspond only to relativistically imaginary or *time-reversed* states of the string (+/-$m \rightarrow$ -/+m), and in this sense c is again seen as the fulcrum of PM exclusion.

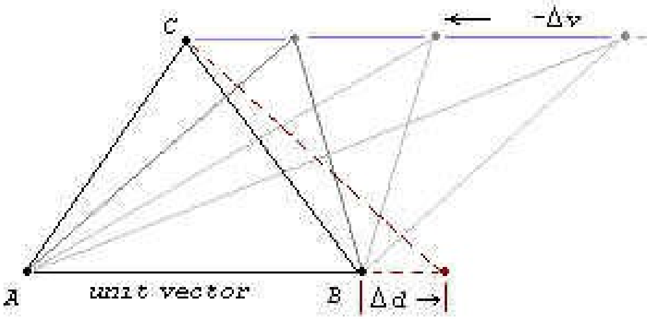


Fig.7

26.) Now suppose that the pair of vertices *A* and *B* in Fig.7 move apart in the manner of 'two electrons' by an increment Δd with velocity *v*, due to a Lorentz dilation in some frame *S* attached to a third 'electron', *C*, *decelerating* from velocity *v*' to *v*" relative to (say) *A*. The total mass of *A* and *B* is a property of the string *AB*. (From the point of view of *A* or *B*, of course, *AB* has *no* mass; the quantity called 'the total mass of *A* and *B*' appears in the system *ABC*.) Because this dilation of distance Δd is in every sense a material expression of relativistic spacetime (Δd represents a changing charge density at *AB* which corresponds to a changing magnetic force on *C*), we conclude that *AB* 'really' does stretch, that real work is done, and that Δd is proportional to an increase ΔT in string tension. Eq.16 tells us that if string tension on *AB* increases by ΔT, then *c* must also increase if mass *m* stays constant. Therefore if *c* is to be constant there must be an increase $\Delta\mu$ in mass-per-unit-length in proportion to the tension.

27.) This could be interpreted from a particle point of view as saying that we take *A* to be 'at rest' and that an 'inertial mass' attached to *B* (call it m_b) increases as *B* recedes from *A* in proportion to $\gamma(m)$. But we cannot measure Δm_b directly; it is a component of a kinetic energy measured at *B* relative to *A* in a certain frame *S*. We could say that because the positive acceleration Δv of *B* relative to *A* is inversely proportional to the *negative* difference *v*'- *v*", then this increase in $m_b = E_0$ will be cancelled out by a *de*crease Δm_{ab} in the total relativistic mass of *AB*, so that the total energy *E* is conserved. Evidently there is a sense in which any variation in a 'rest mass' is actually an abstraction, along with the point-particle to which the notion of 'rest mass' is attached, because what any 'observer' like *C* measures for any given ΔT is just the displacement corresponding to an increment ΔE in the total energy. In PM we cannot give a clear meaning to this increase in terms of an objective property attached to *A* and yet not to *B*. No scalar mass can be 'freed' from this dynamical confinement. Insofar as the interaction $A \leftrightarrow B$ is modelled as a photon exchange we could equally well call ΔE_0 a photon mass (see *Section 2.2*. para.14) if we wish; but if we arrange (in imagination) to put ourselves in the position of *A* or *B* so as to investigate a photon by direct inspection - i.e., we become 'an electron' so as to 'absorb' it - then *AB* becomes null lightlike and vanishes in the act of inspection (i.e. the spacetime interval s^2 and the photon momentum density both go to zero with each act) expressing the fact that the 'inertial mass' vector $+\boldsymbol{m}_i$ is automatically annulled by the antiparallel 'gravitational mass' vector $-\boldsymbol{m}_g$ to give zero mass-energy over all. (This is our restatement of Weber's postulate that the sum of all forces on a particle is zero in all coordinate frames; see *Section 2.2*. para.11.) The force and the photon in *AB* are only *real* for a system including a third point of measurement, *C*. So a variation ΔE_0 in rest mass can only be regarded as virtual, in exactly the same sense in which a photon at rest is unobservable. Or: the work of displacement corresponding to an increment ΔE in the total energy is *an alteration in the state of rest of the unit AB*. (A detailed thermodynamical justification of this point of view is given in *Section 2.4.)*

28.) Remember that the 'electron rest mass' is the quantity E/c^2, which is not an isolable 'thing' but an invariant component of the total kinetic energy in Eq.20. The fact that E_0/c^2 is not isolable from the total energy of *AB* reflects the fact that kinetic energy disappears at rest (by definition), and since a scalar point particle contains zero internal energy its state of 'rest' is just its annihilation, which is why we say that PM mass is vectorial. The components of the total energy in the Hamiltonian of a free scalar particle are not well-defined because a free system has no well-defined state of kinematic rest. In relativity this translates ultimately to the lack of a rigid global boundary condition on spacetime and an uncertainty in defining the gravitational energy of a particle. PM questions the meaning of a scalar particle and offers the Hamiltonian a local habitation inside the boundary condition of each exclusive vectorial dyad, thus effectively eliminating the free energy from the thermodynamic equation by identifying it with the internal energy of a system always in conservative equilibrium. In PM it is inescapable to associate the invariant E_0/c^2 with the string as a whole, for how should we separate one 'end' from the other? Instead of being the mysterious internal energy of a scalar particle which may be *sometimes* at rest, E_0/c^2 becomes an invariant limit on the internal energy of a unit vector which is *always* functionally 'at rest' (by the PM dynamical definition of 'rest'). Relativistic kinematic transformations of this invariant unit vector are improper views.

29.) This dynamical conception of rest inevitably involves both the 'radiation' and 'matter' components of the dyad simultaneously in the PM supersymmetry - the 'ends' of the string together with the connection between its ends. In general, a 'rest energy' can be interpreted as the kinetic energy of internal components whose vector sum of momenta is zero for some class of observers. But if all the energy of *AB* is to be seen as internal energy (because we have eliminated the ambiguous free energy), and if the vector sum of momenta internal to *AB* is to be zero for *all* observers (i.e., $E_0 = constant$ defines 'absolute rest') then the component momenta are the momenta of massless particles oscillating at the speed of light. Considered as a whole the 'electron pair' *A* and *B* confines the virtual momenta $p = E/c$ of photons carrying inertia (virtual mass) $m_\gamma = h\nu/c^2$, and the *total* 'rest mass' of *A* and *B* becomes equivalent to the electromagnetic self-energy of their interaction. So it is precisely the relativistic locality condition introduced with the invariant photon speed *c* which ensures that *E* 'contains' E_0 in sense that E_0 is **in the past of** *E* whenever we make a measurement. (This follows by definition in PM from the identity *unit tension* ≡ *unit time* ≡ *unit length* ≡ *unit speed*.) Again, the fact that 'a photon disappears as soon as it stops travelling at *c*' is seen to represent both the confinement-bonding of *A* and *B* and their dyadic exclusivity as vertical position states in PM.

30.) If a photon disappears as soon as it stops travelling at *c* one might say that the experimental condition for confirming that a photon travels at *c* is never to observe it! In this sense the photon has a unique and indeed rather curious role in a science of measurement. There is a profound issue exposed here: We can in principle make two 'position measurements' on a photon, at points of emission and absorption; the second measurement not only brings it to 'rest' but annihilates it, which means that we need a new photon for any further experiment. It seems possible to go so far as to say that an inability in principle to make an observation on a photon 'in motion' is precisely the essence of what we *mean* by the 'speed of light'. Two positions and associated times can only define a velocity, but three could in principle define an acceleration. The fact that we cannot achieve three points of measurement on the same photon can be seen as the 'reason why' we say that a photon 'cannot accelerate', meaning that it 'travels as fast as is possible all the time'. This is both the definition of the limiting relativistic velocity *c* of a photon whose momentum cannot be transformed away to zero for any observer, and the definition of an elementary object in PM geometry (i.e., a non-Euclidean straight line completely determined by two and *only* two points; see *Section 2.1*.para.4).

31.) This bears some unpacking. It might be objected that scattering of photons, in the Compton effect especially, shows that it *is* possible to make more than two position observations on a photon. But this only underscores the subtlety of the question of continuity of particle identity in quantum theory. In the classical view of the scattering of light by an electron, of course, this issue of continuity of identity would not arise in the first place: Scattering is stimulated re-radiation of a new continuous wave by a charge set in oscillation by an incident wave, and so one could say that precisely *because* the scattered wavelength is exactly the same as the incident wavelength there is no continuity of wave identity implied classically. Paradoxically, it is the very *absence* of continuity of wavelength through the Compton scattering process in quantum theory that then encourages us to imagine a *continuity of particle identity* because the discontinuity is proportional to the Planck action constant. But it is not possible to spatially localise the quanta in a light wave and the mental picture of a 'particle of light' is misleading. True, the Compton effect demonstrates that a photon cannot be 'split'; but since a perfectly monochromatic photon would be a wave train of infinite length which would take an infinite time to reflect from a mirror this is hardly helpful to the conception of a 'particle'. It remains necessary to model scattering in terms of interference. Hard X-ray or gamma ray photons with wavelengths close to the Compton wavelength of the electron are needed, and in a process of intermodulation the photon and electron wave functions are superposed to produce a *new* photon wave-packet. The result is that the incident and scattered photons are *not* 'the same photon', and this is so in an even stronger sense than 'two interacting electrons' are not 'the same electron' for the reason that each of the two photons is 'internally marked' with a different wavelength whereas 'an electron' (as a conventional particle) never is.

32.) In an electron-electron scattering the question of continuity or discontinuity of identity loses meaning in QED: Firstly the Heisenberg relations mean that the two position states cannot be definitely discriminated, and secondly the commutation relations mean that the two electrons cannot be uniquely marked either. (This is natural in PM, remember, because 'two electrons' do *not* have separate identity in the first place, or they each have half of the same identity). In a scattering context the rule that an electron cannot be marked is just what we mean by a constant electron 'rest mass': two electrons come out of the scattering region each with the same invariant equivalent wavelength of 0.51 MeV. On the other hand according to the Compton relation

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{20}$$

an X-ray photon λ' emerging from the Compton scattering region at any non-zero angle θ cannot be *un*marked, because of the constant h. In this sense it is an intrinsic **serial non-identity** of 'two photons' which is at the heart of the statement that a photon does *not* have 'rest mass', whilst an intrinsic **parallel identity** of 'two electrons' is at the heart of the statement that an electron *does* have 'rest mass'. In terms of PM's projective geometry of non-differentiable doublet states this relation becomes very natural.

33.) All of the aforegoing raises questions about what can be meant physically by the constancy of quantities like rest mass and the speed of light in a discrete, relativistic PM theory. We will be able to address these more fully in the light of the thermodynamical statistical considerations of *Section 2.4*. Meanwhile consider the suggestion that in a relativistic theory the meaning of a direct comparison of *c* on two closed trajectories *AB* and *CD* that are parallel-displaced, or on two arms of a scattering trajectory displaced by some angle θ = *BAC*, becomes impossible to define except operationally. When we say that a photon γ of frequency ν on signal line *BA* and another γ'of frequency ν'on *AC* are related *via* a scattering region at *A* we automatically specify *A* as the vertex of a definite angle closed by measurements at *B* and *C*, and a determination of *c* in respect of γ and γ' always involves relativistic specifications of time and distance and mass-energy on three sides of a triangle of fermion positions. The absence of a singular objective state ulterior to the differently transforming views of *A, B, C* and *D* means that comparing measurements is a *process* of preserving a dynamical self-consistency between serial views of a *plural* system, and it is not obvious that a relativistic 'field' theory can be discriminated from a sophisticated renormalisation device for keeping numerical 'constants' constant in terms of one another.

34.) Does *c* really exist 'out there' other than in a formal sense? Yes, but the 'obvious' qualification is that it exists as a constant only for **systems** of dyadic PM units - because the physical objectivity of 'out there' *consists in* the process of self-interaction in such a system. The fact that *c* is dimensionally a speed rather than a pure number tells us that it is intrinsically a *ratio* and has no determinate value *inside BA* or inside *AC*, only *between* them. So the basic structural relation between the elements of a PM system required to produce this 'determination of *c*' will, we infer, always be just the type of structure required for a determination of the ratio of the electrostatic to the electromagnetic unit of force, this being the original physical meaning of the quantity first experimentally derived in 1856 by Weber & Kohlsrauch. A dozen ingenious measurements were made of this ratio during the next thirty or forty years by means of Leyden jars, galvanometers, resistance coils and battery circuits and it was well known that *c*, the reciprocal of the square root of the product of the vacuum permittivity and permeability, was invariant in terms of units of length and time. One especially vivid definition of this ratio was given by Maxwell:

> 'Hence we may define the ratio of the electric units to be a velocity, such that two electrified surfaces moving in the same direction with this velocity, *have no mutual action*. Since this velocity is about 300000 kilometres per second, it is impossible to make the experiment above

described.’

Maxwell was thinking in terms of *c* as a disturbance propagated in a continuous elastic ether, and of course this was completely independent of later relativistic theory; but intriguingly it prefigures the Minkowksi spacetime representation of special relativity in 1908 according to which Lorentz transformations between relatively moving bodies become 4-rotations of systems which are considered to be ‘travelling through spacetime’ uniformly at the speed of light. This conception of an ‘absolute’ ensemble velocity (strictly a speed, a directionless scalar) is in its way no less difficult to interpret physically than is a convective ether. On the other hand, the highly intuitive description of electric and magnetic field interactions in terms of Lorentz-transformations of moving charge-densities made possible by SR seems closer to the spirit of electrodynamics prior to the Maxwellian synthesis. The actual theoretical and practical structure of the acts of measurement involved in determining *c* can be described as precisely that structure of interdependent relativistic self-adjustments required to ensure that the same value of *c* is always generated.

35.) In a certain sense the very essence of relativity is its circularity. It appears that a relativistic ‘field’ theory cannot be discriminated from a sophisticated renormalisation device for keeping numerical ‘constants’ constant in terms of one another, for the very good reason that the theory so marvellously models a renormalisation device which is *the structure of Nature itself*. Einstein stated that he was led to SR largely by the conviction that an electromagnetic field in one frame was nothing but an electrostatic field viewed in a differently moving frame. Today we are more used to hearing the story told in terms of the Michelson-Morley experiment, whose null result certainly makes the idea of a global rest frame in the form of a continuous ether difficult to support; but what Minkowski spacetime puts in its place is too often interpreted as though it were just some subtler kind of ether. Statements such as that ‘the speed of light is the same everywhere’ or that ‘an observer travelling at 0.9*c* would still see a beam of light overtake her at *c*’ perpetuate the misleading notion that making *c* equal in all reference frames is the same as setting some global parameter of a continuous ether. Of course one cannot observe the motion of a light ray in vacuum, and the meaning of ‘everywhere’ is changed by SR in such a way that the mapping of the field of ‘all observers’ onto a continuous substrate requires the mixing-up of space and time and the dislocation of planes of 4-space simultaneity, a process carried to the extreme of singular rupture of the continuum in GR, a geometrical theory whose *entire force* is in the unmeasurability of its fundamental geometrical elements.

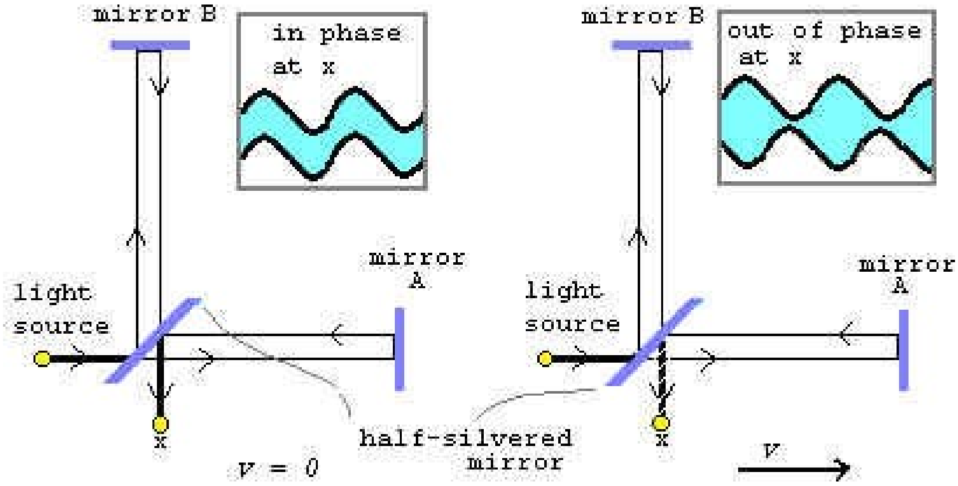


Fig.8. Schematic of the Michelson-Morley apparatus showing expected result for a relative ether velocity. Measurements parallel and transverse to the motion of the earth are compared by rotation of the whole experiment through 90º. If light propagates through a stationary continuous ether medium at *c*, split beams that arrive back at *x* in phase when arm *A* is transverse to the motion (left) should arrive out of phase when arm *A* is turned through 90º (right). They don't. SR concludes that the speed of a light wave is defined only with respect to its source, not with respect to a background medium.

36.) PM proposes that this continuous mapping should be radically broken up and abjures the substrate from the start, accepting that the interlocking conspiracy of relativistic physics works on the more subtle level of *pluralism*. From this point of view the Michelson-Morley experiment (Fig.8) is an apparatus for exhibiting the fact that the phase-locking of clock rates occurs by definition when we stipulate that the rates of the clocks are to be determined by the phase, which follows from assuming a constant identical 'speed of light' on both arms of the interferometer. Asserting this identity is a 'choice' forced upon us by a 'conspiracy' of nature in the sense that the only possible definition of identity is precisely our inability to demonstrate it in direct measurement. The simple fact is that two dyadic exemplars of unit speed are by definition closed to direct inspection by one another. To certify metrical identity by applying one measuring rod directly to another is a process that can only be completed by asserting their ontological identity. But in physics there is at least one electrodynamical interval and an **included angle** that identifies two distinct rods in the first place. That the *essence* of 'measurement' is the *separation* of systems by lightlike intervals is if course the core insight of relativity, and amounts to the assertion that an electrodynamical manipulation of a system that reduces the angle ϕ towards zero in Fig.9 is a process that *would* simultaneously constrain unit lengths α and β to transform to an indiscernible identity if ϕ were ever zero. This is an operational statement that denies the meaning of ontological identity in a plural physics.

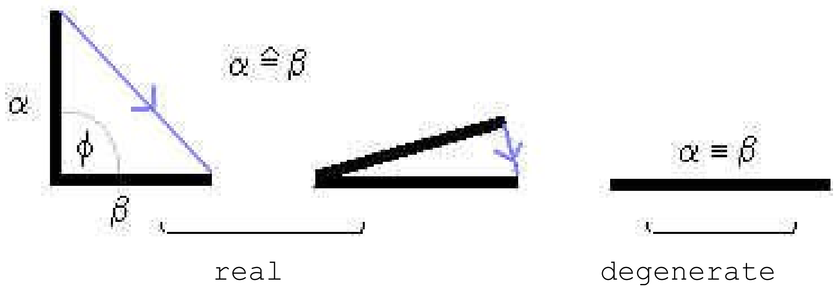


Fig.9. The essence of physical measurement is the scaling algorithm that maps α onto β. The degenerate identity α ≡ β has no metrical meaning because it is not an operation.

37.) Consider the *gedanken* experiment of **Einstein's box**: A tubular box has a spring at one end *A* and a sticky pad, *B*, at the other. A ball is fired by the spring from *A* and sticks on the pad at *B*. Like a Mexican 'jumping bean' the box recoils against the release of the spring and moves in an opposite direction to the movement of the ball, until the arrival at the ball at the far end cancels the momentum and stops the box. Knowing the distance through which the box has shifted, the travel of the ball from *A* to *B,* and the total mass, one can work out the mass of the ball which has been transferred from *A* to *B*. One finds that although the box has moved in the direction of *A* the centre of mass has not. (See Fig.10) Similarly, from the shift of a light-box due to a photon flash and Maxwell's theory of light one can work out, for a constant speed of light, that there is a mass transfer from *A* to *B* equal to the energy of the flash divided by c^2. This result holds for any observer of the box and for any similarly constructed box of any length.

38.) But how in practice do we measure the movement of the box to determine where the centre of mass is? If we have a single box suspended in some fixed laboratory reference frame the meaning of the experiment is plain. But now consider that an experimental event is schematically represented by a chain of such 'closed' boxes, interlinked in such a way that a transfer of mass-energy in *AB* is able to trigger a flash in *CD*, and so on. Make the system entirely self-contained with no background 'gravitational field' for a reference frame. Moreover let the 'rigid' frameworks of the boxes be dissolved, leaving pairs of contiguous 'ends' each subject interdependently to one another (Fig.11). Analogously, each of our PM dyads represents such a closed Einstein's box, closed in the sense that we can form no conception of seeing into it 'sideways on', or of placing a mark on it externally, and there is no frame of reference other than may be provided by some configuration of other boxes. Since we can only examine each closed box by its ends, and because this examination can only take the form of registering the states of adjacent boxes, we have to question the force of the claim that *c* is a global constant with the 'same' value inside *AB* as it has inside (say) *XY*.

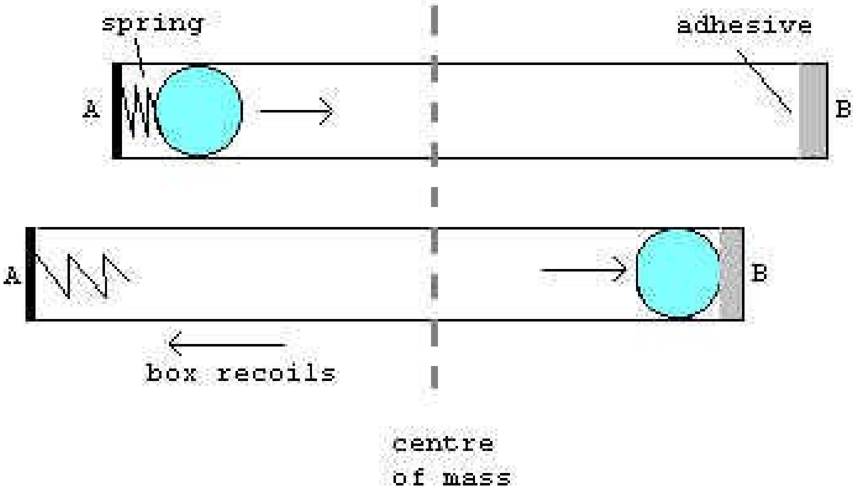


Fig.10. Einstein's box. Transfer of energy from *A* to *B* by a photon, as by the ball, results in recoil of the box. If momentum is conserved the centre of mass in an isolated system cannot spontaneously move, so this proves that the energy of radiation carries inertial mass.

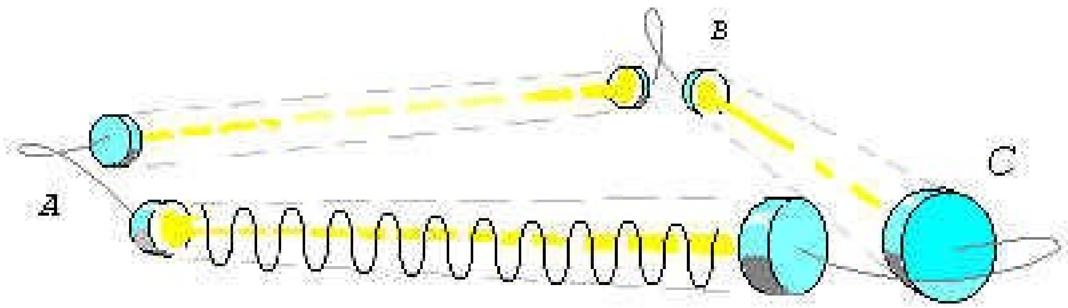


Fig.11. Schematic idea of a system of interconnected 'Einstein's boxes', observationally connected to one another only *via* their ends. As an isolated system this is a relativistic 'three-body problem' with only emergent solutions for variables of mass, length and time where *c* is not an externally-fixed absolute. A 'constant speed of light' can be seen as a normalisation parameter, representing a self-consistent *equilibrium* condition for the system. From this point of view a state of agreement on a normative value of *c* stands for the system attractor of highest photon *entropy*.

39.) Because we cannot 'look inside' a box our confidence evidently reduces to the operational fact that a subsystem of boxes called an 'observer' gets consistency in her 'measurements' by using the same value of *c* to relate other subsystems (or 'objects') to herself and to one another. Assuming that *c* 'really does' transfer from box to box in some absolute ontological sense can be seen to be equivalent to the assumption that *unit scale* transfers intact underneath a Lorentz transformation of *unit distance*. But does *c* or unit scale 'actually' transfer with objective constancy in this way? The relativist must answer: 'The point is, how could we possibly know? The meaning of relativity is that *no further reduction by metrical inspection is possible*.' Indeed so; but for that very reason SR tells us nothing directly about the presence or absence of absolute qualities, only that they cannot be measured (by definition of measurement as ratio), so this fails to get to the heart of issue - which is that rational comparison depends on a prior condition: The possibility of *distinction*. Relativity itself cannot supply this primitive condition, which is the plurality of the world exemplified in the Pauli exclusion principle. So the theory of PM postulates just such an 'absolute' quality of distinction in its primitive geometry and extends the exclusion principle accordingly to an indefinite series of 'quantum numbers' associated with each of the doubly-connected position states of PM space. And now, because this absolute object is a discrete *a priori* structure in PM space, instead of a value recovered in the limit of a continuum of distances and times in smooth classical space, we have no interest in its quantitative 'constancy'. Indeed its essential characteristic is that it should be the very antithesis of a transferable metrical quantity.

40.) SR itself strains towards the same perspective. In denying the possibility of the globally fixed frame of reference in which such a transference of an absolute comparator might have meaning, it *requires* the introduction of a *local* absolute whose comparison has no meaning. Due to its classical origins the theory is structured in such a way that time, distance and mass-energy conspire in order to preserve the idea of the constancy of *c* in a continuous vacuum. But, of course, as soon as SR is generalised to 'include gravity' the first thing GR demands is that we replace the idea of the constancy of *c* in vacuum with a route-dependent variable corresponding to curved lightlike geodesics. This is done by what Eddington famously described as a 'put-up job', a circular conspiracy of nature in which no measuring rod can be applied which doesn't share the spacetime geometry of the rod to be measured, and so the unique 'constancy' of *c* in fact consists in the prohibition of meaningful comparison of *c* with itself in the system of nature. So we see that in a clear, if subtle, sense the value of *c* not only *might be* arbitrary in respect of its underlying *discrete* function but, if relativity is radically true, *must be*.

41.) In the end we realise that inside a spacetime theory *it does not matter* whether or not the *c* in one measurement is the 'same' as the *c* in the next because we can give no clear meaning to the question in terms of spacetime relations of observables. But the fact that it does not matter is itself the key to a meta-relativistic physical principle: The crux of Einstein's insight appears to be that *c* becomes a **constant of varying norm** and the reason that this is possible consistently with experience and relativity theory can be seen to be that the variation in norm of *c* is *not a continuous* *differentiable function of spacetime interval for any possible* ***pair*** *of measurements*. In a sense, of course, this is 'merely obvious' because, as we have argued, it is only the same as saying that photons cannot accelerate (or more exactly, never exceed the speed of light) in 4-space. But the true meaning of *why* they cannot is only obvious when understood in the context of a many-centred space structure like that of PM, where an 'absolute' acceleration can be represented as a rotation of a primitive unit vector in a space of complex planes (see *Section 2.5*), and where a complete, simple graph without loops or simultaneous multiple edges supplies the *boundary condition* that enables this variation to be discrete.

42.) One interpretation of this system of discrete norms is that c is an extremum of a gauge invariance Θ which would exist for any (isolated) set of PM elements at an arbitrary *common* value of Θ, in the sense that only $\Delta\Theta$ has physical significance rather than any absolute value of Θ itself. In other words, as suggested (Fig.11), $c = const.$ can be regarded as a normalisation parameter for a system in underlying thermodynamic equilibrium. In this sense $\Delta c = \Delta\Theta$ represents a degree of actual disorder which is the negation of the condition $c = const.$, and insofar as c embodies the spacetime structure this allows us to label 'gravitational' entropy as *positive* in harmony with the positive entropy of thermal systems. A c which is a constant of varying norm thus represents the disorder of the gravitational field, which is mysteriously missing from the homogeneous and isotropic space of GR.

42.) Cosmologically, this can be seen to be equivalent to addressing the flatness problem by means of a type of discrete VSL (variable speed of light) theory, but a theory in which the spacetime 'origin' is a vacuum equilibrium, a flat sheet in PM state space rather than a singularity, recovered by a subtraction over Δc on all lightlike paths. In the standard FLRW-type cosmological models based on modifications of GR the idea is to devolve various quantum fields back to a cosmic $t = 0$ in order to reconcile gravitational negentropy and thermal entropy, matter and radiation. Our interpretation on the other hand is based on a geometry designed to exclude singular degeneracy and depends on the idea that the entropies of the 'radiation' and 'matter' components of a PM system are *always* in underlying thermodynamic and supersymmetric equilibrium. This is the issue to which we now turn.

2.4 consistency with quantum thermodynamics

1.) We showed in *Section 2.3* that the meaning of relativistic rest mass as an invariant is naturally associated in PM with a minimum condition of the energy of the 'string' as a whole. The kinematical meaning of the term 'rest' now disappears into a conservative zero-sum of the kinetic energy $+\Delta K$ and the potential energy $-\Delta U$ of the two displaced ends of the string. In other words it describes a stationary condition of the Lagrangian function of the interval *AB*. This doesn't of course mean that the string as a whole takes on a ballistic role as a species of 'free particle'; in PM space the concept of such a free kinematic object has altogether disappeared.

2.) The function of the term 'rest' is now that of an essentially dynamical limit of the transformation

$$\Delta E = \frac{T}{\sqrt{1 - \frac{v^2}{c^2}}} - T = \Delta T \tag{21}$$

which we will say is just the relativistic fluctuation in a total *internal energy* ($E \propto$ tension T) where a lower bound E_0 represents a limit on a transformable *free energy* available for 'inspection' (mutual work of inter-transformation) among a system of dyadic strings. This equivalence between mechanical mass-energy and thermodynamical energy depends on the system being *reversible* and on the possibility of representing mass-energy as heat; but, assuming (for the moment) that we can demonstrate these conditions, it is straight away evident that E_0 represents only an extremal value of this thermodynamical limit. The only value of E_0 to which we ever have access is a ratio, since the notion of 'weighing' an absolute mass in isolation has no meaning. So in principle this extremum occurs at any arbitrary *common* value of E, because only ΔE has physical significance rather than the absolute value of E itself. In terms of such a general dynamical definition we are able to say that a fermion dyad and the photons it 'contains' are jointly and equally 'at rest' in a system whenever the free energy F vanishes, that is when

$$F = E - T_K S = 0 \tag{22}$$

where E is the mean energy, T_K is the absolute temperature[45] and S is the entropy. It is a basic thermodynamic principle that for any T_K a system spontaneously seeks to minimise the quantity F, and δF becomes zero at equilibrium. Because T_K is essentially independent of E and S it happens that at high T_K the system minimises F by maximising S, whilst at low T_K it does so by minimising

45 We adopt T_K for the temperature to avoid confusion with T = string tension.

E. In PM we are able to say that *any* dyad has minimal free energy and is therefore at dynamical rest in a system which is always in underlying equilibrium, and the reason why this does not imply an homogeneous and isotropic universe in PM is to be found in the scale-free supersymmetric geometry of the theory.

3.) The semi-classical spacetime view from which the basic Planck-Einstein quantum condition was developed is obviously very different, because equilibrium in that case does imply homogeneity. There is a thermodynamically favoured expectation that the universe should be in equilibrium, in conflict with observation on moderate scales easily accessible to us where plainly it is not at all homogeneous but rather has elaborate structure. This leads to the need to resolve the conflict by recovering far-from-equilibrium complexity as a mesoscale condition from a theoretical presumption of homogeneity at the extremes of micro- and/or macroscales, which extremes are considered to embody the smooth simplicity of a 'fundamental' state from which the universe has devolved.[46] Microcosmically this 'classical' view assumes the continuity of a smooth spacetime background which can be approximated by averaging particle actions over smaller and smaller scales. Macrocosmically the same presumption is implemented by the technique of averaging energy-density variations over larger and larger volume scales until, at some cosmic scale called the 'homogeneity scale', it is supposed to become possible to treat galaxy clusters as small energy-density fluctuations in a smooth fluid.

4.) This procedure ratifies as valid the application of GR to cosmological problems. But the procedure does not guarantee that the smooth spacetime field modelled in this way is other than an *effective* field, produced by applying a 'normalising factor' to some topologically more intricate structure. Quantum gravity theories admit the likelihood of a flaw in the microscale implementation of this homogeneity principle and seek a discrete structure specific to very small scales; but they never consider the possibility of a flaw in the assumption of macroscale homogeneity.[47] This is because of the success of GR of course; but GR is a classical continuum theory that predates quantum theory, and this is reason to suspect that attempts to quantise 'spacetime' are attempts to quantise a normalised effective field that is only a shadow projection of the underlying structure. PM is even handed, by contrast, insofar as scale *per se* is to be emergent and the underlying discrete structure of spacetime will have a *scale-free* representation. Spatial homogeneity and isotropy, then,

46 It is interesting to speculate that there are historical reasons for this perspective. The birth of mechanics took place in the context of a pre-scientific mediaeval religious cosmography, according to which earth was nested at the centre of a series of increasingly-perfected celestial spheres culminating in the outermost unblemished crystal sphere of heaven. It may be that lingering echoes of the dogma of superlunary perfection, refracted through Galilean and Newtonian mechanics, remain undamped even today.

47 There are astronomers and cosmologists who do, based on observational evidence of fractal dimensionality in galaxy distributions (see *Section 2.6*); but they are a few, precisely because there is no plausible connection between fractal clustering and the standard physical models based on GR.

are not fundamental principles in PM, but local variations in energy density map to an underlying *nonlocal* equilibrium whose dyadic units are connections in a scale-free complete graph.

5.) In the traditional view two interpermeating 'gases' of point-electrons and point-photons (schematically speaking[48]) are each far from equilibrium in nature, constrained into highly ordered structures with generally small entropy, and special conditions are specified - such as those of cavity radiation - to investigate the behaviour of radiation in equilibrium with matter. In the case of black-body radiation in equilibrium with an ordinary cavity its entropy is considered to rise to a maximum value for a given energy. Crudely speaking, the photons are maximally disordered analogously to thermal particles and F is therefore minimised for large E by Eq.22. Classical spacetime theory locates a condition of actual equilibrium in the cosmological past, when spacetime itself had the character of a black body cavity prior to the decoupling of the CMB. The essence of PM is to propose that there is an underlying non-continuum point of view from which radiation is *always* in equilibrium with matter and that *any* structure of PM dyads has the character of a black body cavity, independently of emergent 3-geometry.

6.) An illuminating way of looking at this is from the point of view of the definition of entropy, S. Entropy is often thought of as a measure of disorder, but the meaning of order is ambiguous and difficult to quantify. Perhaps more fundamentally S measures the proportion of a system's energy unavailable to do work. Thus by Eq.22 if a 'gas' of photons totally reflected inside a classical cavity (in vacuum) were to be thought of as a system of kinetic particles with uniformly the highest possible thermal speed, then F will be minimised by way of the system seeking the largest possible S.[49] This large entropy will signify the maximum of some quantity measuring 'energy unavailable for thermodynamic work'. Now in general the unavailable quantity is heat; but *this is also just the definition of 'rest energy'*, a quantity of untransformable internal energy locked in as E_0. The question then arises as to why photons are absolutely massless and, in a closed cavity, make their energy unavailable as completely thermalised high entropy radiation, when thermodynamically speaking they could equally well make it unavailable in the form of mass and low thermal velocities.

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48 Actually the distribution of photon momenta $p = E/c = h\nu/c$ given by the Planck-Einstein quantisation condition can only be considered to become particle-like throughout the cavity at high frequencies; in general the wave modes have to be taken as filling the cavity at all wavelengths to properly represent the probability. Nevertheless, the emission and absorption probability distribution *is* in general particulate (energy $h\nu$ goes wholly and uniquely from one electron to another) and, given a vacuum cavity, in the 'Wien part' of the Planck law where the constant h becomes significant the probability distribution is obviously isomorphic to the electron distribution of the cavity wall for S_{max}.

49 'Particles' with zero inertial mass are obliged to bounce around at the maximum possible velocity all the time. From this 'Brownian motion' point of view we can connect the radiation entropy with an equal probability of 'finding' the same *speed* of radiation at any possible point of measurement. Given $c = constant$ this speed field contains zero negentropic order.

7.) A standard answer to the subtle question 'why do photons have zero rest mass?' is that they are required to do so as vectors of an infinite range electromagnetic gauge field. But this answer is really an equation in several unknowns - the meanings of 'absolute' rest; the absolute speed of light; 'range' in an infinite 4-volume; the fact that the electromagnetic action is not exclusively local[50] - and thus it depends on some unknown physics, not least the origin of inertia. We can gain some insight into the question from PM's point of view by returning to the fact that according to the equation $E = mc^2$ and the thought-experiment of 'Einstein's box' (see *Section 2.3*) energy carries inertial mass, and so according to GR it must also couple to the gravitational field. The mass of a box of thermal radiation at rest therefore includes a certain contribution due to the photons it contains, and an ensemble of such boxes will gravitate towards one another on a steeper gradient than will a similar ensemble of boxes containing no radiation. Each box is entitled to regard itself as at rest, and its contents are a part of its rest mass-energy. Therefore in a spacetime representation there is an important difference between radiation considered as internal energy or as external energy, reflecting the fact that 'rest' and 'mass' are *co-dependent* properties of the underlying topological structure:- closed geometrodynamical 'loops', or the general class of structures replicating the closure of triads in PM state space. We will have much more to say about this later and in *Section 2.5.*; but here we note that the masslessness of 'the photon' reduces to the principle that in general no *pair* of photons may have *the same* determinate 4-momentum, inasmuch as the definition of rest mass is inherently an extremal equilibrium condition of a *bounded ensemble* of momenta, and we see here the familiar PM exclusion principle requiring the serial non-identity of photon pairs as the obverse of the parallel identity of electron pairs (see *Section 2.3* para.32).

8.) The quantity which is minimised internally by a confined thermal gas of particulate photons in accordance with Eq.22 is Δp, the fluctuation in mean momentum density, leading to large S and vanishing F. That is, the high entropy of cavity radiation means that there is no momentum density gradient across any part of the phase-space volume (on some cell scale which becomes the basis of the classical quantum statistics) available for *internal* work, and this applies for any arbitrary total mean energy E because only Δp is physically significant and Δp - for practical purposes - is zero. However the same E does become significant when cavity α is brought up to another cavity β with a uniform energy $E' = E \pm \delta E$, because now δE reappears as a potential gradient that may do *external* work on α and β. When both 'boxes of radiation' remain at rest in the doing of it we say that they gravitationally attract one another.

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50 Specifically, experimental effects of the Aharanov-Bohm type show that an electron's phase may be affected by the variation of a magnetic field even if it is in a region from which the magnetic field is *completely* excluded.

9.) So we find that radiation *does* have a scalar mass charge or 'rest mass' for the purposes of GR when considered as the internal energy U of a closed system; and for an ensemble of such closed systems - which we may then consider to be at rest in a $T_K << m$ radiation bath - the Helmholtz free energy F of the ensemble is minimised collectively, in proportion to the inertial masses of its members, where that inertia represents, equivalently, both unavailable thermal *internal* energy (high S) and minimised *external* kinetic energy (low E). This describes a system of massive particles each being a composite of individually massless particles. The fact that we can characterise the same particles in these different ways depending on emergent space relations suggests that it would be consistent to extend the co-dependency of 'rest' and 'mass' proposed in para.7 to the implication that *rest mass* and *volumetric scale* are similarly co-dependent on a more fundamental topological property of those relations.

10.) This approach is inevitable from the point of view of PM's geometrical definitions. It is attractive because the volumetric quantum field approach leads to divergences as a result of not being well defined in the limits. For example, we can suppose this new ensemble to be a free-falling inertial system of spherical cavities contained in a larger volume, an ideal gas at minimum kinetic temperature all of whose free energy is locked in the inertial mass of a system of 'particles' at rest - small ΔE, small S. This is a new thermodynamic phase of the system, call it phase 2, in which the thermodynamic contribution of phase 1 assumes the status of an unmeasurable internal energy-per-particle, U_{unit}, equivalent to the 'mechanical' or 'bare' mass, m_{mech}. This quantity m_{mech} can be completely arbitrary, but all that is measurable is the experimental mass m_{exp} corresponding to the energy that the particles have when their energy of interaction ΔE is non-zero (interaction ≡ measurement), which incorporates a term $\delta m \approx \Delta E$, so that $m_{exp} = m_{mech} + \delta m$. The meaning of an invariant rest energy then appears as an ill-defined quantity which can only be treated by being effectively eliminated by a vacuum renormalisation, leaving an experimental mass which is not an invariant. It is not invariant because it depends on a relativistic interaction energy - for example, on the electromagnetic interaction energy of one 'particle' with another - which involves variables that are intrinsic to the process of measurement itself. This process is just the *process of existence* of the particle.

11.) The analogue of such a renormalisation process in QED when carried out covariantly for all observers leads to a way of formally sweeping under the carpet a quantity that cannot be measured; in effect it sets m_{mech} infinitely *negative*, by hand, so as to cancel an infinite *positive* quantity δm coming from the electron's interaction with the quantised field and with itself. This leads to doubts about whether renormalisation can be regarded as a mathematically valid procedure and whether

QED is a fully consistent theory. However the infinite δm comes in the first place from quantising a continuous field over an indefinite spacetime volume where there is no limit on the virtual particle proliferation that contributes to the 'phase 2' experimental mass m_{exp}. This is equivalent to being unable to set an upper bound on the temperature of the (vacuum) radiation bath because of an enclosing infinity of nested 'cavity' volumes, which can be assumed to cancel against a mirroring infinity of unmeasurable internal phase-1 mass-energy. But if infinite renormalisation is mathematically invalid this obviously doesn't mean that renormalisation theory itself is *physically* invalid. Renormalisation is the physical *essence* of the quantity 'mass'. The question is how a cancellation of unmeasurable quantities can be recovered as a finite natural relation.

12.) We can view renormalisation as a process of attempting to realise an equivalence between mechanical energy and thermodynamical energy in the form of an equilbrium constant called 'rest mass', an equivalence which as we noted earlier is realisable only for a system that is reversible. But the 4-space representations of natural processes are *not* reversible, or are in only some instances approximately reversible, which is a statement of the Second Law of thermodynamics. In thermodynamical terms 'rest mass' represents an extremum of entropy, as we have seen, a maximum of energy unavailable for work; but the Second Law states that maximum entropy is only completely determined inside a closed system. This is frustrating for a spacetime representation because of the causal time-asymmetry dictated by the expanding spherical wave fronts. There are no truly reversible closed regions of spacetime: the past light cone of every observer is locally closed, but it is not reversible; likewise the sum of all past light cones of all possible observers. Only a phase volume equal to the whole of spacetime contained in the past *and* future light cones of all observers can be regarded as both closed and possibly reversible in principle. But this volume is not really available to any observer; the sum is not local.

13.) So this is why the best solution to mass renormalisation in QED is to be found by summing over in the limit of all possible *virtual* spacetime trajectories, leading to what is in effect a heat bath of infinite temperature. A heat bath of indeterminate but finite temperature cannot be calculated with in a spacetime representation because it implies a system either not truly reversible or not truly closed; there is then no natural relation between the internal mechanical mass-energy of the particles (phase 1) and the thermodynamical energy of the system (phase 2). However the infinity of energy states contributing to δm in the sum over all possible virtual spacetime paths relieves us of the responsibilities of having to calculate δm or of having to interpret what is, in an irreversible system, an *unnatural* relation between δm and m_{mech}. Declaring both unknowns to be infinite means that δm has an infinite probability of endothermically reversing an infinite disorder incurred in the work done exothermically by an infinite internal mechanical energy m_{mech}.

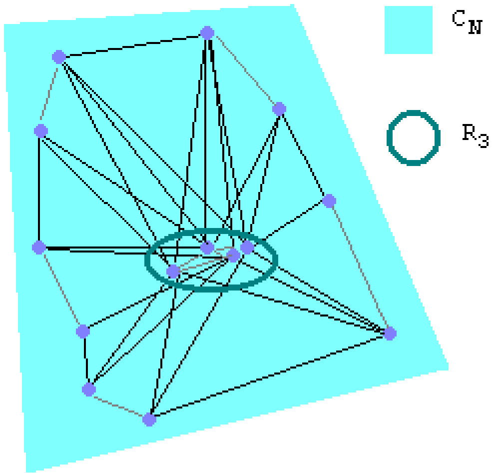
14.) In this way all values of m_{exp} can be made to lie on one critical surface in an infinite-dimensional coupling space. The effect of this is equivalent to ensuring that absolutely any experimental rest mass whatsoever signifies an extremum of entropy which is a constant for a system in equilibrium. But although defining an equilibrium condition in this way permits perturbative calculation to proceed beyond the first term, it is ill-defined in principle, and this is the problem with the renormalisability of QED. An infinite-dimensional coupling surface can be said to have all possible topologies. Without a finite boundary condition any point on the sheet can be regarded as coupled to any other point *and to itself* in an infinite number of arbitrary ways. Or in other words this coupling space can be said to attain a critical-point dynamic equilibrium for m_{exp}, but only because an infinite spectrum of values of the *correlation length* are granted equivalence.

15.) So we see that the appearance and the cancellation of infinities in renormalised QED are obverse sides of the same pathology connected with correlation 'lengths' that appear as infinitesimals, loops and multiple edges, because the absence of a finite scale factor and the absence of any topological constraint go hand in hand. In a properly consistent theory where the equilibrium function of mass can be well-defined in principle it would be necessary to constrain the topology of the coupling space to remove infinitesimals, loops and multiple edges, and we recognise this as a description of three of the founding definitions of our PM state space, where proper reversibility is preserved on dyads of all scales. In other words the infinite-dimensional coupling space of QED and PM state space are approaching the same conception from the directions of continuity and discreteness respectively, but the top-down continuum approach imports a semi-classical pathology. The finite PM state that we have begun to build bottom-up can be described as being a critical surface in the sense of an *effective* 2-space due to the *reduced dimensionality* of a hyperspace of N dimensions where the ratio of 'correlation length' to 'lattice spacing' is always just unity on any dimension irrespective of emergent metrical scale (i.e., the 'lattice' is not metrically regular of course; it is a scale-free graph where *all* couplings are effectively 'nearest neighbour couplings'). As we will see in more detail in *Section 2.5*, each of these N objects is itself actually a complex self-orthogonal 2-manifold whose proper null state superposes $+t$ and $-t$ improper representations indiscernibly. This is the origin of a *proper* reversibility underlying the spacetime representation with its emergent *im*proper *ir*reversibility, as noted at the end of *Section 2.3*.

16.) PM's discrete projective pre-geometry thus gives the closure and reversibility needed for consistently interpreting mechanical mass-energy as 'heat' in an N-dimensional equilibrating phase space. It does this by abandoning the idea of mass as a scalar property and substituting the idea of a vector resultant, innate to elementary dyads, which is only ever non-zero in improper (i.e. local spacetime) system representations. In terms of thermodynamics this means that equivalent equilibria occur at both extremes of internal kinetic energy and are independent of the absolute temperature; in terms of string modes it means that the fundamental mode of all partial harmonic modes, and the resultant of all partial modes, are views of the *same* mode, in the one case deconstructed analytically, in the other constructed synthetically. The absolute value of both Fourier transforms in the *proper* state is the same - zero. The absolute value of both transforms in the *improper* state is again the same, but now it is unity, the unstable fixed point of the renormalisation group transformation. The dyad acquires *unit scale* in the elementary measurement system of an equilateral triad; but it is still a reversible superposition of +*t* and -*t* and so it remains possible in this case to say that the appearance of an invariant phase-1 E_0 is coemergent with the maximum-entropy equilibrium of phase 2. If the triad of dyads is 'three electrons', then a 'bare' or mechanical mass is just the inverse of the 'electromagnetic mass', or the dressing, and the antiparallel vector resultant remains zero. Here the phase transitions occur not as upper/lower, inner/outer state boundaries in a differentiable space, but on a nonlocally non-differentiable path around a closed loop, yielding an isentropic adiabatic relativistic zero-mass as a stationary state of a dynamical equilibrium between oppositely propagating vectors in a linear sequence of running constants renormalised at unity.

17.) In general, δm and m_{mech} are identifiable with the free and internal energies. On any network path these quantities are properly cancelled against one another, reversible-closed-dyad by reversible-closed-dyad, like $m_i - m_g = 0$. Fermionic 'matter' and bosonic 'radiation' preserve a context-dependent supersymmetric ambiguity on such a path, which can be traced in terms of creation and annihilation of virtual photons and electrons. But these modes separate out in the 4-space representation, where $m_i - m_g \neq 0$. This is because what 4-space represents is a structure of intersecting paths. It is a state function corresponding to all possible different path functions in the state space defined in relation to some emergent kinematical zero-point of inertial 'rest'. In PM the path function is fundamental to the state function, and the breaks in the complex path are boundary conditions of 2-manifold phases that interpenetrate linearly without regard to scale (the nonlocal subset of all pairs of electron states) instead of being nested volumetrically dependently on scale (the local subset of all individual electron states). In other words the set of states $\mathbf{R}_3$ lying on a 3-surface exists at the intersection of *N* members of a universal set $\mathbf{C}_N$ of complex 2-manifolds (Fig 13).[51]

51 We shall use be using the conventions $\mathbf{R}_3$ and $\mathbf{R}_4$ quite loosely,. Emergent $\mathbf{R}_3$ is to be the Euclidean 3-manifold of classical mechanics, $\mathbf{R}_4$ the Lorenztian 4-manifold of relativistic mechanics which is to *include* both the flat Minkowski metric space of SR (denoted $\mathbf{R}^{1,3}$, M^4 or simply M) and the pseudo-Riemannian non-flat metric tensor space of GR.



The surface defined by the set of states $\mathbf{R}_3$ exists at the intersection of *N* members of a universal set $\mathbf{C}_N$ of complex 2-manifolds. The local space is nonlocally context-dependent within the projective space of $\mathbf{C}_N$.

The hyperspace $\mathbf{C}_N$ is a complete graph, but for clarity only a few of the 2-manifolds connecting points of measurement in the state space are shown.

Fig. 13

18.) So it is because the renormalised mass phase is fundamentally a reiterative path function in $\mathbf{C}_N$ and only emergently a local state function in $\mathbf{R}_3$ that the renormalisation of phase at a 'particle' surface in $\mathbf{R}_3$ is not well-defined. And to characterise the mass of confined radiation, instead of saying that radiation contained in a closed cavity volume of radius *r* is 'really' composed of massless phase-1 vector bosons with infinite range, but has an *effective* phase-2 rest mass *m*, we can speak of the mass m_γ of vector bosons confined on linear 'flux tubes' of a gauge field of *N* broken phases of arbitrary range Δx.

19.) Here *any* value of relativistic scale associated with Δx always defines an absolute unit scale associated with a pair of mutually-cancelled changes of vectorial mass phase. The emergent relativistic symmetry of the system describes it as a local kinetic system of monadic mass-points analogous to a far-from-equilibrium thermal gas, but underlying this symmetry is a nonlocal system of stationary states in equilibrium where all dyads are massless at dynamical rest. The mass-point picture deals in state functions; but a photon is massless because it is intrinsically a path function of a system, not a state function. This is only to say, as we did in *Section 2.3*, that dyadic confinement by no more and no less than two measurements is why a photon is 'massless' and can't be accelerated (in the 'vacuum' of this cavity) and why to 'observe' a photon is to annihilate it - photon energy *is pure work* and in a sense it never existed because this work is the expression of its equilibrium with electrons by complete confinement to the supersymmetric PM 'cavity'. It is, as we have seen, actually *one aspect* of the thermodynamic equation of state of the PM dyad and represents (for the simple electrostatic case) the displacement work done on *A* and *B* by one another (as measured at C, of course), which is emergent as the Coulomb repulsion due to what we call charge.

20.) In PM the underlying equilibrium state of cavity radiation is a constant path function of *any* system of charges in $\mathbf{C}_N$, which is emergently recovered or imitated in the form of a state function for certain $\mathbf{R}_3$ configurations of charges. We deny that there is *any* probability of 'finding a photon' except where there is some measurement of the state of a charge, which means at one or other end of a string joining two electron energy levels (free or bound). The photon momentum $p_\gamma = E/c$ is an excited state of a string, the square of its 'rest' momentum $m = E/c^2$, which, given $E = h\nu$, is the angular frequency or 'pulsatance' of a stationary wave equivalent to a 'photon mass' $m_\gamma = h\nu/c$. This represents a departure from an ideal extremal equilibrium (supersymmetric mass $m_i - m_g = 0$) to a real dynamical equilibrium where $m_i - m_g \neq 0$, corresponding to non-zero Lorentz forces. The point is that the equilibrium is preserved for these arbitrary force transformations by the vertical renormalisation of the string because (as suggested in *Section 2.3*), c = *constant* is a normalisation parameter for a system in underlying thermodynamic equilibrium. A non-zero probability density of photon distribution remains confined to the boundary conditions of the network of n_γ lines connecting all charges in the cavity wall and will thus vary with the wall's electron density n_e like $n_\gamma = n_e(n_e - 1)/2$. So in a sense we resurrect Planck's conviction that the quantum of radiation action is related to the quantum of charge and must be a property of the resonators: We say that the entropy of the radiation behaves statistically like that of a system of independent particles because the resonators are *not* independent particles. Instead of taking Einstein's route, which rather than quantising the resonators leads from $\hbar$ to the quantising of a set of interacting fields, we bring the quantisation condition back to stationary conditions of supersymmetric dyadic resonators - PM strings.

21.) PM supersymmetry gives us this dual view: A spacetime theory, which is the reciprocal implication of a free point particle theory, breaks an underlying supersymmetry belonging to a linear network. Paths on this network are geodesics of a non-real-differentiable *hypersurface* - a 2-space in the sense of reduced dimensionality of a critical-point system with a correlation length always equal to any 3rd dimension. Normally we would say that such criticality is characteristic of a low-temperature phase where electron F is minimised through the minimising of E, with quantum behaviour emerging close to absolute zero. But this (projective) 2-surface carries the contracted representation of the world *as mapped by photons*, and can be thought of as analogous to a 'high T_K

crystallisation phase' of matter where all proper distances are normalised to null unit distance. From the point of view of this mapping, E becomes completely arbitrary because of the vanishing of ΔE - of all *fluctuations* in fermion energy - as measured from a zero-point vacuum energy renormalised (for relativistic invariance) at infinity.[52] One could say that an identical *maximum boson entropy* condition obtains for any ensemble of photons whatsoever, because a complete uniformity of relative speed (c) identifies all possible universal configurations of charges as equivalent 'cavities' of *minimum fermion entropy*, and that this is why the real meaning of radiation in equilibrium - touching again on the quantisation issue in PM - is to be found in bridging the divide between the physics of fermion and boson such that apparent $S_{\max}$ cancels against apparent $S_{\min}$. We can already see from the above that the key to this bridge in PM is again the fact that c becomes a constant of varying norm.

22.) In PM because of 'photon confinement' we can say that a supersymmetric equilibrium is conserved in each dyad. High T_K photons maximise their entropy S as an absolutely orderless 'gas' with c = *constant*, no part of which is free energy F available for *internal* work, exactly to the extent that the photons are confined always by low T_K fermions and can thus contribute an internal energy U which is the rest mass E_0 of a 'pair of electrons'. Conversely, then, this pairing is rather rigidly ordered in the sense that its large locked-in energy m_e represents the same quantity U in a form which is unavailable (chemically) for *external* work. No work at all is done overall, in the sense that all work is *virtual* work (see later) of longitudinal scale transformation, a renormalisation of the doublet zero-point state of gravitational rest. The low T_K 'electron pairs' which carry this large locked-in energy E_0 seek to minimise F by minimising their kinetic energy ΔE; and to the extent that ΔE is just the displacement work done by the photon energy U this is why we can say that an electron's mass-charge *resists* Coulomb repulsion due to its electric charge with an exactly equal and opposite inertial force: The electric charge and the donated photon mass are one and the same thing, and that thing - the dyadic gravitational mass - is the vectorial inverse of the dyadic inertial mass. Inertia is thermodynamical. Looked at differently, the relativistic increase in electron inertial mass with increasing v is a response to increasing kinetic energy E - coming from energy of electrodynamic acceleration - by tying up an increasing fraction of the Helmholtz free energy F as entropy, in the limit of $S = m_e = \infty$ at c. So for fermion and boson considered together, the free

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52 In other words, the spectrum of particle masses is a hierarchy of ΔE associated with different phase changes in the $\mathbf{R}_3$ vacuum expressing the same entropy in terms of different T_K and different characteristic length-scales, with the standard deviation of the distribution curve of length-scales increasing as the peak value of T_K decreases. The resemblance of this pattern to the experimental black body law described by the Planck distribution (and also qualitatively by the Maxwell-Boltzman ideal gas velocity distribution) is not accidental.

system-energy F is minimised overall.[53]

23.) If the momentum density of a virtual photon 'gas' and a relativistic electron mass can be thought of as dual representations of the PM supersymmetry then the increase in 'inertial mass' with Lorentz contraction of the 'cavity' AB at increasing relative velocity is analogous to an approximately adiabatic compression of an ideal gas. At low rates of compression the internal energy changes in balance with the rate of heat flow from the environment, or in other words the mass lies approximately on a Newtonian isoinertial curve. As the rate of compression increases, however, the heat transfer lags behind the internal energy so that the rising curve of the mass point on the 'pV diagram' in Fig.14 cuts with increasing steepness across successive curves of constant inertial mass. Such an adiabatic process is generally isentropic, if it is reversible; and indeed any process like a displacement in AB is *individually* reversible. So the proportionality between mass and entropy suggests that the reason why processes involving multiple AB-like systems are *not* reversible is connected with the fact that mass is an emergent property only of transverse vectors in larger-than-triadic systems in PM.

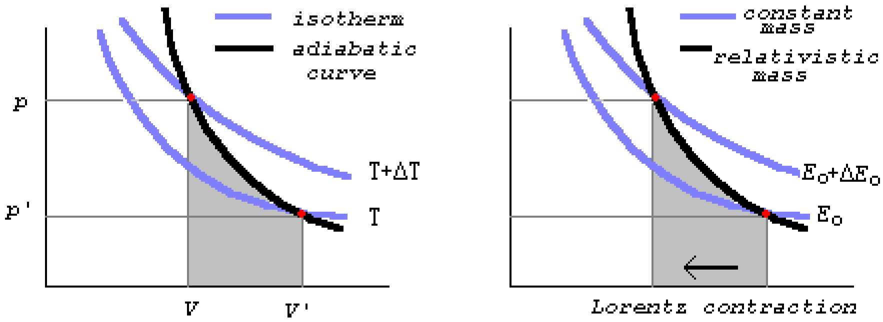

Fig.14. Analogy between approximately adiabatic compression of an ideal gas and relativistic contraction of PM 'cavity'. The inertial mass behaves like the photon energy density of an isolated system.

24.) The 'end-on' aspect of an abstractly isolated single unit vector has no associated properties of mass or scale. Only in its photon representation does AB behave like a perfectly reversible isentropic, isothermal ideal gas where $\Delta U = 0$ and $Q = W$. This is equivalent to saying that the photon density of the system AB is an *isolated* ideal gas, or that being an ideal gas its internal energy U is dependent solely on a temperature which is by definition always as high as possible (the kinetic energy of a fixed density of confined photons all moving at c cannot change). But it is also

53 From a spacetime point of view this is why a single electron isolated in an empty universe could not radiate, having (in the sense of the Wheeler-Feynman theory) no absorber: The Helmholtz free energy of a radiation field open to infinity has no upper bound.

valid to say in these terms that any heat entering the gas has to leave it immediately as work, keeping the internal entropy (photon 'rest mass') constant (at zero) independently of any change in 'pressure'. Therefore we can have the case of a quasi-isolated system which behaves like a cyclic heat engine, a periodic system with some rate of positive or negative work due to a *throughput* of energy *exchanged* with adjacent systems.

25.) In this view an external source of energy may assume an equivalent role to that of an internal non-zero photon 'rest energy'. If we imagine a closed network of coupled systems, a triad of resonant oscillators *AB, BC, CA*, then if they are free to do so they will tend to an equilibrium condition where three values of T_K are the same and overall no work of displacement is done, but the equilibrium energy is arbitrary; *there is no way to determine an absolute internal energy of this isolated system*. The only way to determine a characteristic energy of *ABC* is to bring it up to another network *DEF* and measure a potential difference; otherwise, its arbitrary equilibrium energy will be the zero-point energy. This is the same as being unable to measure an absolute internal energy *U* in a gauge theory where only ΔU has physical meaning. Extending this to the whole of PM space we can imagine that such an unknown equilibrium condition exists as the ground state for a complicated network of many semi-autonomous interacting processes where entropy will in general not sum locally to zero over subsystems like *ABC*. In this network photon momenta become subsumed in a system of potentials coming from fermion mass constraints, and an ideal gas model gives way to critical-point complexity. The scale-free correlations of PM supersymmetry represent this critical-point behaviour, embedding self-organising islands of emergent local stability on a surface of dual phase in the state space.

26.) If the dynamical meaning of 'rest' refers to thermodynamic equilibrium, the universal closed set of dyads must be assumed to be in underlying equilibrium in the sense that *kinetic* realisation of 'absolute rest' occurs (in principle) at extremes of global temperature. The low temperature global equilibrium is a minimum of energy $\Delta E_{set} = 0$, locally approximated when electromagnetic gauge symmetry breaks in a superconductor below T_{crit} and photons' 'range' is restricted to the surface by acquiring a 'rest mass'. Kinetic rest is here realised for a fluid lattice of 'electron pairs' whose relative speeds are uniformly zero (Cooper pairs). The magnetic field is expelled completely from the conductor, which is a way of saying that the photons take away the mass shed by the electrons from the inside of the conductor. Loops of the photon mass-gauge field now run around the outside but none cut the surface of the superconductor to terminate on the massless pseudo-bosons moving in it. According to PM the 'cutting off' of the Cooper pairs from this outside gauge field[54] now

54 The 'cutting off' occurs only in $\mathbf{R}_4$, of course, as a breaking of Lorentz symmetry. The underlying topology in $\mathbf{C}_N$ remains that of a complete graph.

makes overt the *complete photon confinement* ‘inside’ them which is the supersymmetric state of PM symmetry. (In a sense these objects are analogous to isolated ‘magnetic monopoles’ of the PM gauge, despite their doublet position basis.[55] The efficiency of superconductivity should be proportional to the completeness of this divorce of the current from the encircling gauge loops. See *Section 2.5*.) Inversely, global equilibrium at high temperature would be a maximum of entropy, which is equivalent to all electrons attaining relative rest by uniformly moving at the highest thermal speed possible, or at c. That is to say, at c all electrons realise their underlying dyadic nature, all possible pairs being Lorentz-contracted isomorphically to the null intervals of their own photon bonds, effectively coming to ‘rest’ as a null field of massive vector bosons. In this way the absolute zero of temperature mirrors an absolute maximum of temperature, opposite limits corresponding to thermal speeds of c and $-c$, and the breaking of electromagnetic gauge symmetry occurring at either extreme realises the virtual photon mass of the supersymmetric PM dyad.

27.) The point is that PM forces us to generalise these extreme thermal equilibria and say that not only extremal conditions of the dyad but *all* intermediate conditions of the dyad are at dynamical rest, in the form of scale-free stationary states. What allows us to do this is the elimination of real free energy. All kinematics become equilibrium-conserving transforms of unit scale, and in a sense all displacements become *virtual displacements,* referring dynamics back to statics according to d’Alembert’s principle, such that the very existence of a non-zero limit on untransformable internal energy (i.e., the existence of a rest energy E_0) arises from a conservative equilibrium of applied and constraining forces.

28.) Determining constants E_0 and c of the relativistic energy in a given dyad, AB, is analogous to setting the absolute internal energy $U_{unit} = const.$, precisely for the reason that these quantities cannot be directly known. In the normal way, U is assumed to be a constant of any free-expanding system (such as AB in isolation, free from any constraint); but the components of a Lagrangian function $L = K - V$ for confined virtual-mass vector particles (kinetic + potential energy of photons internal to AB) cannot directly be seen. Thus to state that U cannot be directly measured simply restates the *definition* of U as a constant of the system, in the sense of $c = const.$: That is, U can only be deconstructed by inspection into separate components K and V of the total Lagrangian by a process of de-isolation which *changes* U. This means that the significant quantity is ΔU or the

55 In fact *because of* the doublet basis state. We can understand this intuitively from the point of view of the conventional symmetry principle leading us to expect monopoles of magnetic point charge by analogy with electric point charge monopoles. Such magnetic monopoles have never been detected. Our inverted resolution of the puzzle can be expressed in this way: It is not that free point magnetic monopoles do exist, but that (in the PM space of quantum basis states) free point electric charge monopoles *do not*. The difference between positive and negative charge is entirely contextual in $\mathbf{R}_4$ and that charge exists at all depends on the *exclusionary* unit objects in $\mathbf{C}_N$ whose reciprocal implication is photon confinement.

change in U, which *can* be measured (because its change *is* the process of 'measurement'), by the application of constraint due to adjacent similar systems. In this 'measurement' process (interaction) we have $\Delta U = Q - W$, or the change in internal energy of *AB* equals 'heat absorbed' from the environment of *AB* minus 'work done' by *AB* on its environment. Since 'the environment' is the set of other systems like *AB* interacting with it, where the total energy of the set $U_{\mathrm{set}} = Q_{\mathrm{set}} + W_{\mathrm{set}} =$ *const*., we can see that heat exchanged and the work exchanged are different names for the same quantity measured as energy transfers with different sign, so that energy changes ΔE in each system like *AB* sum to a constant quantity E_{set}. This conserves total energy and so satisfies the First Law of Thermodynamics for the set.

29.) If the set is in perfect isothermal equilibrium then the work done on each system in the set is obviously identical to the heat supplied in each system in the set, or $Q \equiv W = 0 = \Delta U$ for any arbitrary temperature of the set. If the set is not only in equilibrium but completely isolated (i.e., the finite universal set of systems like *AB* is self-contained) then the total heat supplied is zero and the 'electron temperature' of PM space is zero by definition, whatever the 'photon temperature' corresponding to the constant mean internal energy of each individual system. Absolute zero therefore defines an equilibrium condition which is not automatically the state of lowest energy density. In the sense that this zero-point temperature of the set is completely independent of its 'absolute scale' and of the internal energy U of its component systems like *AB*, we can apply a 'virtual' energy increment ΔU which is equivalent to a quantity of 'virtual work' done in a 'virtual free expansion' of the set, without changing the thermodynamic equilibrium.

30.) This can be likened to a global increase in a scalar field energy functionally equivalent to an 'inflation' and in those terms would be equivalent to varying c inside a global relativistic symmetry in a VSL-type theory. In PM, however, the notion of a unique history of a global vacuum that occurs in GR is an abstraction; it is, on the contrary, the locality and pluralism of SR that paradoxically gives physical meaning to Δc and thereby to the complete complex causal structure involving both 'subluminal' deterministic histories and 'superluminal' scale-free nonlocal correlations (see *Section 2.2*). The set of N systems is analogous both to a PM quantum 'field' with N particle states continually fluctuating around an average value (a self-driven resonance around the network), and to a *set of quantum fields*, like N 'Higgs fields' each associated with a different breaking of the PM supersymmetry, each yielding a different mass vector with a different phase of c. In this unusual sense the analogue of 'inflation' can be said to be driven by the 'fluctuating value of the Higgs fields'. The thermodynamics of PM supersymmetry are such that both the smoothness due to scale-free correlations, *and* the anisothermal, anisotropic structures of local measurement, are coemergent aspects of an overall isentropic reversible system where $\Delta S = 0$.

31.) In the emergence of real energy differences ΔU between systems in the set, real work is done so that in individual systems $\Delta U = Q - W \neq 0$, although $\Delta E_{set} = 0$. But according to PM the important point is that this work always represents the appearance of an *improper* distinction between two components of the internal energy - the heat energy ΔU and the free energy, F, the portion available for transformation to do external work. The emergence of this distinction corresponds to the *exclusively* ***improper*** *emergence* of 'electron mass' in thermodynamic disequilibrium, because the electron mass *is* this distinction, or $m_e = \Delta m = m_g - m_i$. But simultaneously the *exclusively* ***proper*** *cancellation* of this distinction in the null massless photon representation expresses the conservation of a supersymmetric equilibrium in each dyad.

32.) This emergent distinction of ΔU from ΔF is relativistically improper in the sense that it belongs not to the unit system (the PM dyad) but to the embedding set of systems constituting the environment on which work is done; neither is it a property of that global embedding set inasmuch as there is no embedding meta-set constituting an external environment for it to do work on. In other words neither an individual dyad *AB* nor the universe as a whole has an intrinsic real mass because both are *closed* sets: The former minimal set is abstracted from its embedding, whilst the latter maximal set negates the meaning of embedding. One way of expressing this is to say that both these extremal sets recover *absolute* mass $|m|$, which is always just indeterminate m_{unit}, whereas real determinate mass Δm belongs to embedded multiplets where $m_g - m_i \neq 0$. If we think about this we understand in a new way how the *essence* of mass in relativity is in fact its pluralistic nonlocality.

33.) That the distinction between components of the system energy corresponds to the emergence of a distinction between m_g and m_i has a clear formal basis in quantum theory and classical gravity. That is, the *internal energy U* corresponds to the Lagrangian of component kinetic and potential energies coupling to the energy-momentum tensor $T_{\mu\nu}$, so we can say that $\Delta U = \Delta m_g$, or the gravitational mass-energy. However the *free energy F* corresponds to the Hamiltonian energy function for the work done against external generalised momentum coordinates, $\Delta F = \Delta m_i$. These are components of a mass shift which can be shown[56] to occur due to finite-temperature radiative corrections to the mass of an electron in a heat bath of photons of temperature T_k much smaller than

56 John F Donoghue and Barry R Holstein, *Aristotle was right: heavier objects fall faster,* Eur. J. Phys. 8 (1987) 105-113.

the electron mass-energy m_e. The two components (of the shift δm) are related like $\delta m_i - \delta m_g = 0$. The two total masses are identically m at $T_k = 0$, but at $T_k \neq 0$ then

$$m_g = m_i - T_k(\delta/\delta T_k)m_\beta = m_i - 2m_\beta \quad (23)$$

where m_β is the radiative mass correction. But correction to what? What is m? In PM, the only answer is that m_β is a correction to a 'unit' mass or absolute mass $|m|$ with *no* determinate value at all, which is a scale-free null-vectorial property of all 'electron *pairs*' and has no meaning as an isolated scalar quantity. So a 'measured' quantity of mass must by definition be entirely such a 'correction', emergent in the interaction of a system of such 'pairs' (the minimal system being the PM triad) and proportional to a departure from perfect thermodynamic equilibrium.

34.) At equilibrium $T_k = 0$ we say that the electron mass vanishes, as exemplified in the vanishing of inertial mass in the 'broken electromagnetic gauge symmetry' of superconductivity or the superfluid regime. The Cooper pairing reveals this phase as a special case of the general PM principle, which we see not as breaking a symmetry but as *repairing* the PM vectorial supersymmetry in which m_g and m_i cancel one another away. Conventionally, even though the strong equivalence principle enshrined in GR states the scalar identity $m_g \equiv m_i$, there is *no* profound theoretical demand that gravitational mass should vanish together with inertial mass. GR cannot allow it to vanish because gravitation is required to be a field coupling with a scalar mass charge equal to the total energy. Only if the total energy vanished could the mass charge coupling go to zero, and energy conservation is required by time displacement symmetry everywhere on the manifold continuum in GR. But from our point of view m_g must indeed vanish along with m_i at $T_k = 0$, and this goes hand in hand with the time-*reversal* symmetry on the *discrete* PM dyad.[57] We propose that a continuum field-coupling type of theory like GR cannot apply to the limiting equilibrium states of *simple* systems of small N (irrespective of scale), where the linear relation between the mass-energy and the gravitational potential, which in GR applies for off-equilibrium systems of large N, breaks down. (See *Section 2.6*)

35.) The null cancellation $\boldsymbol{m}_i - \boldsymbol{m}_g = 0$ represents the *renormalisation* of Eq.23 above for the case $\delta m_i - \delta m_g = 0$, $T_k = 0$, which is what we have deduced for the case of PM equilibrium. Our point of view must be that all of the effective electron mass is a 'radiative correction' to an unmeasurable internal

57 Energy conservation is the principle associated with time-displacement symmetry in spacetime theories governed by Noether's theorem, just as momentum conservation is associated with space displacement. However it is well known that the theorem doesn't apply in the case of time-reversal symmetries.

energy U of a dyad which is just $U = E_{unit}$, and that m only ever has physical significance as $\Delta E \propto \delta m$ when $T_k \neq 0$. The reason why $m_e \neq 0$ can thus be expressed equivalently as: *(a)* because perfect equilibrium does not obtain; *(b)* because there are always photons in the equation; or *(c)* because the speed of light has a determinate value - which is the obverse of a perfect equilibrium condition in which a 'ratio' of degenerate values of c could have no meaning. (The emergence of non-degenerate phases of c is discussed in more detail in *Section 2.5*.)

36.) We can compare our model with the canonical 'particle in a box' where $\Delta E \propto \delta m$ is a property of the box, not of the particle, in the sense that it can be seen *either* as a contribution due to photons emitted and absorbed by the charge on the particle, *or* as a shift ΔE_p in the energy of each of the zero-point radiation modes of the box due to the presence of the particle,

$$\Delta E_p = \frac{1}{2}\omega_p\left(\frac{1}{n(\omega_p)} - 1\right). \qquad (24)$$

From this latter point of view ΔE can be thought of as proportional to a certain 'refractive index'

$$n(\omega) = 1 + \frac{2\pi}{\omega^2} f(\omega) \qquad (25)$$

where $f(\omega)$ is the forward scattering amplitude for a photon of energy ω. Here $n(\omega)$ is associated with the altered matter distribution in the box due to the presence of the particle, and is proportional to the particle's 'bare' mass m before the radiative correction δm - i.e., m has to be brought in as it were from outside the experiment. But the problem is that the unrenormalised electron self-energy in QED is infinite, which means that ΔE blows up. In PM on the other hand the unrenormalised dyad self-energy is zero, because the renormalising 'radiative correction' which alone *generates* a non-zero mass is a consequence of the 'experiment' performed by nature itself on the 'box' of the PM dyad.

37.) This experiment consists in the spontaneous emergence of local Lorentz symmetry which transforms dynamically on top of a nonlocal equilibrium condition in which energy conservation is completely undefined. On the underlying graph time-reversal invariance is preserved where all energy states are interchangeable and $\Delta E = 0$ in the absence of coupling. Or in other words the basis state of local position-momentum coupling is not fundamental; it is a transformation of the underlying nonlocal basis state. It represents the transition from time-reversal invariance to time-reversal non-invariance, which is the same as the appearance of thermodynamic potentials. The thermodynamic ‘direction’ of time thus has a local projection in a sequence of states of $\mathbf{R}_3$, but this is only the shadow of a path function in $\mathbf{C}_N$.

38.) The unique direction of *logical* time for coupled systems is a universal, since the universality is just the criterion we use to *define* a ‘direction’ of time. This is thermodynamic time, based on an emergent state phase of the system where an evolving surface divides the phase space into inner/outer, earlier/later, smaller/larger - the topology of $\mathbf{R}_4$ which assigns positive/negative values to a cosmic time. But in PM this projection is relatively inessential. Just as the analogous logical invariance of spin-up/spin-down does not derive from a physical universality of spin direction (i.e., there is no cosmic up/down to which spin states may be referred in $\mathbf{R}_3$) so there is no *physical* universality of cosmic time belonging to $\mathbf{R}_4$ from which ‘microscopically reversible’ values of $+t$ and $-t$ acquire an absolute orientation. On the contrary, $\mathbf{R}_4$ time-reversal non-invariance is a secondary emergent property incurred in what we might call a second-order process of renormalisation of *states* of $\mathbf{R}_3$ (in 6-dimensional position-momentum phase space) each representing the nonlocal folding of *all paths*, which are first-order renormalisations in $\mathbf{C}_{N.}$[58]

39.) In $\mathbf{C}_N$ the *un*renormalised self-energy on every dyad is conserved at zero, because only in a path (or state) renormalisation does energy emerge as a meaningful quantity, and the absolute null entropy actualises both the maximum and minimum force-free extrema of the relativistic case, i.e., we completely realise Weber’s principle (“*the sum of all forces on a particle is zero in all coordinate frames*”; see *Section 2.2* para.11) as a path function of the system in $\mathbf{C}_N$, but *not* as a state function of the system in $\mathbf{R}_3$ where there is no time-reversal invariant equilibration on routes. Where non-invariance supervenes then operator orderings become significant (see *Section 2.5*); paths contribute differently, and interference of amplitudes gives $\Delta E \neq 0 \propto F(C)$, where $F(C)$ is an **electromagnetic flux** through a region or contour C identifiable as some surface in $\mathbf{R}_3$. The broken supersymmetry on $\mathbf{R}_3$ states locally dissociates PM dyads into electron and photon states. For photons the new dynamical minimum is realised around a loop that relates *successive* states at the same ‘point of space’, or at the same vertex, but displaced in *time*; whereas for electrons this minimum is realisable for measurements at pairs of *simultaneous* states, or at different vertices displaced in *space*.[59] This is of course the relation embodied in the standard gauge theory of quantum electrodynamics:

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58 This point of view is very reminiscent of Bohm’s ‘implicate order’.

59 This refers us back to the contrast between the *serial non-identity* of photons and the *parallel identity* of electrons brought out in *Section 2.2*.

40.) Consider that the *rate of change* of phase of the wave function of a neutral particle is a quantity belonging to a series of measurements on a spacetime path. It is governed by the relation

$$\theta_2 - \theta_1 = \left(\frac{\hbar}{m}\right)v \tag{26}$$

where v is the particle velocity. This change connects definite 'points in spacetime', i.e. a relationship between phases of the wave function at the start and end of a spacial displacement. Why does this apply *neither* to an uncharged boson like a photon *nor* to a charged fermion like an electron?

41.) Evidently for a massless particle Eq.26 would predict an infinite phase shift which is not physically meaningful, so to make it applicable to a photon would require a photon to acquire a mass. On the other hand an electron obeys the different relation

$$\theta_2 - \theta_1 = \left[\left(\frac{\hbar}{m}\right)v\right] + \frac{e}{\hbar}F(C) \tag{27}$$

which is the rate of change of phase from Eq.26 plus an added factor proportional to the electromagnetic flux $\propto \Delta E$. This electromagnetic flux, F, which is known from the Aharanov-Bohm effect to represent a potential that acts nonlocally on the electron phase, is thus in some sense equivalent to a photon mass. We can see that this follows naturally from the PM geometrodynamical supersymmetry introduced in *Section 2.3* and on this picture the 'number of lines of force' threading the contour C will correspond to some finite definite number of strings 'cutting' an abstract surface in $\mathbf{R}_3$, whilst the A-B effect signifies the underlying connectivity of the complete graph in $\mathbf{C}_N$ recovered in the approach to low-temperature equililibrium at T_{CRIT}.

42.) In quantum theory it is true to say that the 'charge on the electron' *is* its (probability of emitting and absorbing) virtual photons. In PM the degrees of freedom of the field are restricted to *exchanges*; there are no empty states and the boundary condition is isomorphic to all possible pairings of vertical position measurements, not at real infinity. That the neutrality of the photon is another name for what we call the charge of a pair of electrons is already 'obvious' in the sense that the lightlike 4-vector is the vector of the Coulomb force; in the Feynman representation it is also obvious that the same neutrality represents a time-reversal symmetry under exchange of electron/positron labels on the spacetime diagram (photon and anti-photon being always self-orthogonal). But this picture is only fully realised from the point of view of PM's dyadic *structural supersymmetry*, where we see that the 'masslessness' of the photon (or anti-photon) is another name for what we call the 'mass' (vectorially annulled) of a 'pair of electrons (or positrons)'.

43.) The closest we can get experimentally to realising this structure in a theoretical extremal condition where ΔE vanishes is the superconducting regime close to 0°K, where thermal isolation of a system approximates this theoretical isolated equilibrium and exposes PM supersymmetry in the form of pseudo-bosonic inertia-free pairings of ordinarily repulsive charges. In this case it is possible to say conventionally that breaking of the electromagnetic gauge symmetry causes photons to acquire mass whilst electrons lose their mass inside a region where the electromagnetic flux $F(C)$ vanishes. Inside this quasi-isolated region we have the elimination of relative acceleration between coupled charges with the concomitant elimination of their *internal* field. We say that the region inside the expelled magnetic field of a superconductor is an approximation to an ideal $\mathbf{C}_N$ path function in terms of states in $\mathbf{R}_3$.

44.) Such an ideal state would exist in the isolated equilibrium triad we have already described, where the rate of change of phase around the system would correspond to a neutral massless 'superon'. But this system remains a pure path with no coupling, each dyad being isomorphic to a string of unit scale in an underlying graph where time-reversal invariance is preserved, where all energy states are interchangeable and where $\Delta E = 0$. This complete, simple triadic graph is a limit case of the set of all possible complete simple graphs, and going from this extremum to the general case corresponds (we conjecture; see *Section 2.5*) to the reduction of the quantum state vector *via* decoherence. From the point of view of PM extremal coherence expresses a latency of two phases of the same supersymmetric dyadic object, phases which, with spontaneous thermal symmetry-breaking, separate out in interaction, and the limit of low temperature equilibrium will not be a statical but a dynamical equilibrium superposition of these phases. The two coexisting phases of a system of Cooper pairs are characterised by a strong interaction with *external* magnetic fields[60] and the entire absence of *internal* magnetic fields. The same superposition of phases can be identified with the coexistence of two forms of the mass m in Eq.23 which is already recognised in the conventional quantum formalism. These forms are the gravitational mass m_g and the inertial mass m_i (which of course are none other than our antiparallel PM mass vectors). The former is considered to couple to the energy-momentum tensor and corresponds to the *internal energy* which in our

60 Even though no external flux lines cut the surface of the conductor in $\mathbf{R}_3$!

supersymmetric theory is a quantity belonging to the photon (pseudo-fermion) representation; the latter is the Hamiltonian mass-energy corresponding to the *free energy* which belongs, we say, to the electron (pseudo-boson) representation.

45.) But it is important to emphasise here that according to PM the 'elimination' of the internal mass field is just the recovery of an intrinsic default state, because non-zero mass is an emergent, off-equilibrium *system* property. The external mass identified with the free energy continues to quantify a strong coupling of the system of dyads as a whole with the *external* network, because that is where the system's free energy is. The system can be said to have minimised its free energy by donating it to its photons, then eliminated it entirely by expelling them - in the very particular sense that there is no available gauge loop *within the system* by which a transported photon phase vector can be brought 'back to itself' with a spin of one (which is a possibility only generated by states of 'itself' represented in an electron displacement; see below). The system has only photon annihilation operators and no creation operators. Any available internal loop produces an identical phase shift of zero in the electron 'condensate', which is not a spin-one photon state. Picturesquely, we can say that the line of force or the 'electromagnetic flux tube' of PM photon confinement has been expelled from the dyad. So this is the meaning of the fact that the magnetic flux is expelled to the surface of the superconductor, and the externalisation of mass explains the reason why we can still describe the dyad as a *massive boson* in terms of a relation like Eq.23: It retains mass in terms of its participation in the total inertia which its parent system represents in the network, even though its internal mass-energy is considered to vanish (i.e., to realise its zero-point energy).[61]

46.) If each dyad is always a properly null path element in the phase space of $\mathbf{C}_N$ independently of its state-renormalised potential in $\mathbf{R}_3$ then in $\mathbf{C}_N$ we have equipartition of energy $\Delta E = 0$ in phase-space cells of 'constant (null) size', according to Liouville's theorem; but, because the distribution is real scale-free and equilibrium occurs at an extremum which is a null stationary state, there is no black-body uv catastrophe in $\mathbf{C}_N$. The function of what we call the 'quantum condition' for $\mathbf{R}_3$ thus appears in the mapping from $\mathbf{C}_N$ to $\mathbf{R}_3$, and corresponds to this inversion: That the *true* position-momentum space for radiation is the space $\mathbf{C}_N$ of doubly-connected path elements, not the volumetric phase space of singular point states in $\mathbf{R}_3$. Cavity confinement is a path function of lines in PM, not a state function of points, and the infinite self-energy and uv catastrophe are avoided by the same finite renormalisation.

61 This is obviously analogous to the way in which the individual component momenta of a system of thermal particles can be treated as zero if the vector sum of momenta can be treated as zero. In this case we ought to say that zero-point fluctuations in the resultant mass vectors of individual dyads sum at any 'instant' to the zero-point mass energy of the whole, which is a dynamical vacuum equilibrium.

47.) Position states in $\mathbf{C}_N$ are minimally defined by pairs of particles in $\mathbf{R}_3$. Conventionally, the number of intermediate states contributing to the self-energy of a *half*-pair would normally be infinity minus one, the one Pauli-excluded transition corresponding to the state of the other half-pair. An infinite renormalisation is needed. (The same occurs in a classical theory, where the infinite energy of self-interaction of a point particle with its own field, due to e^2/r where $r \to 0$, appears proportionally to the acceleration in its equation of motion.) But taking the $2N$ half-pairs always together in PM we get N doubly-connected null-vectorial (but not zero scalar) position states, which are all 'volume elements' of the state space, at rest regardless of the continuous d'Alembertian virtual displacements going on due to relativistic scale transformations. These position states exhaust the degrees of freedom of a finite graph with no loops or multiple edges; consequently the sum over any proper path in $\mathbf{C}_N$ is zero, and since the renormalised improper 'self-energy' of any dyad is calculated over a finite number of intermediate transitions in $\mathbf{C}_N$ the total energy remains finite in any numerically smaller region of $\mathbf{R}_3$. Because the 'bare' theoretical (i.e., proper) energy of an isolated dyad is always zero, we can say (*a*) that the electromagnetic energy of every dyad is its energy of electromagnetic interaction, and (*b*) that the inertial mass of every dyad is its energy of gravitational interaction. There is no isolated self-energy or bare mass. Radiation and mass are ensemble properties.

48.) Remember that each dyad is a stationary condition, an extremal resultant of all possible path functions in $\mathbf{C}_N$, so its zero-point energy represents a zero of *potential* in $\mathbf{C}_N$. But this does not imply an absolute zero of energy in $\mathbf{R}_3$. The 'end-on' aspect of an abstractly isolated single unit vector *AB* has no associated properties of mass or scale, but this means that only in its photon representation does *AB* behave like a perfectly reversible isentropic, isothermal ideal gas where $\Delta U = 0$ and $Q = W$. As earlier argued, the network achieves for every dyad the approximate dynamical equivalent of ideal isolation in $\mathbf{C}_N$ in the form of a stationary state where heat entering the 'gas' has to leave it immediately as work, keeping the internal entropy (photon 'rest mass') constant (at zero) independently of any change in 'pressure'. *AB* is thus a quasi-isolated system which actually behaves like a cyclic heat engine, a stationary state of an underlying periodic system which we can identify with a cancelled null rate of positive and negative work due to local-coupling-free time-reversal invariance preserved on a proper path in $\mathbf{C}_N$, a *throughput* of positive-going and negative-going amplitudes exchanged with adjacent dyads, which becomes a time-reversal non-invariant potential in $\mathbf{R}_3$. We can say that the 'open' string mode in Fig.15 represents the nonlocal scale-free coupling which is recovered when local scale-specific coupling is removed.

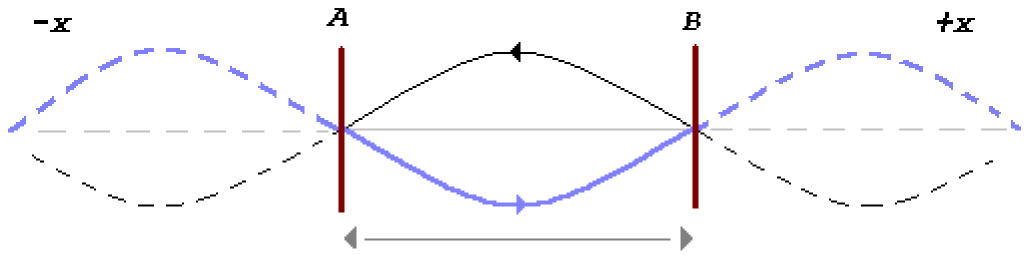


Fig.15. Ambiguity of *AB* as both closed and open stationary conditions. The fundamental mode of the 'closed' string *AB* is a driven resonance in terms of the 'open' string -*x* ↔ +*x*.

49.) Each state's energy is a 'correction' to its own zero-point energy appearing nonlocally in its relations with other states. Each dyadic *m* can be regarded as a correction to an *absolute* longitudinal mass $|m| = 0$, or as the residue of an imperfect cancellation of two arbitrarily large but finite near-identities m_i - m_g appearing with the coupling. So for all real improper Lorentz-transformed views, i.e. transverse views, we have a string tension *T* (see *Section 2.3*)

$$T \propto E \geq 0$$

and hence the 'photon momentum' $p = E/c$ is a quantity which can never be transformed away to zero for any observer. This fact in SR is the reason why vacuum pair-production of an electron and a positron from a photon is not spontaneous but requires the involvement of a mediator particle in order to satisfy momentum conservation.

50.) In SR the energy of a photon track equal to or greater than $2m_ec^2$ can go into generating a diverging electron-positron pair whose combined linear momenta can always be cancelled away by being made equal and opposite in some observer frame. Because photon momentum can never be transformed away, this is only possible if the 'residual' momentum is taken up by an additional 'sleeping partner' in the interaction (say another electron or typically a more massive proton). So a boson cannot just spontaneously 'decay' into two free fermions; the two new points of fermion measurement are 'anchored' by momentum conservation to a third. In PM this translates as the rule that a string cannot be separated into two free 'ends', and again the PM exclusion principle is implicit in the forbidden proper transformation of a massless, pure lightlike, longitudinal field component to a massy, space-timelike component. Only a displaced *pair* of points of measurement can *conjointly* embody this transform to a transverse field component, *im*properly, *in relation to a* *third*. As already mentioned, the included angle is integral to the physics: The process of measurement of mass is fundamentally triadic, because the quantity only begins to acquire meaning along with the elementary unit of relational position in PM, which is the triangle of vectors.

51.) Obviously the (broken) symmetry which permits this superposition of proper and improper views is cognate with the transformation of the *time* in SR, which allows an arbitrary inertial observer S (velocity v) and an observer in another frame S' comoving with a dyad AB (velocity $-v$ relative to S) to disagree about the length of AB because their own clock rates differ by a factor proportional to $\gamma(v)$. But note that although S' is comoving with AB it is *essential* to the very notion of transformation in SR that the proper clock time in S' is *not* a proper clock time of AB itself, measured at A or at B; rather it is another third-party measurement. SR defines 'measurement' as a third-party transverse operation, which is why AB, as self-observed in isolation, does not contain a real clock at all. It is a null lightlike interval which only 'contains' an imaginary projective time; real time is a function of the structure of AB's environment; and this again expresses one of the aspects of its dual character as a *longitudinal* component of PM space, the null and scale-free or 'rigid' PM unit vector underlying transformations of *transverse* scale.

52.) The geometrical interpretation is that the PM exhaustive-connectivity rule for a bounded complete graph (*Section 2.1.* para.4) requires that two arms of any angle cannot be open to infinity but must always be closed by a third. However you attempt to evade this rule and *treat* vertices as free monopoles, theoretically 'collecting two mass-points together' to get $2m$, you are always collecting the two vertices joining *at least three strings* together. This must mean that the quantity $E = 2mc^2$ is an unstable energy configuration belonging to this 'second-order perturbation' term involving at least three 'particles' of total rest energy $E \geq 3mc^2$. In the maximum-entropy condition of supersymmetric equilibrium, where $\Delta E = 0$, the equivalent mass of each of three dyads is indiscernibly just m, or the group is symmetrical in m under exchanges of all *pairs* of measurement coordinates. The perfect antiparallelism of self-orthogonal vectors in each 2-manifold produces no tensor gradient and $+t \cdot -t = 0$.

53.) In this limit $|m|$ is properly *and* improperly zero (or rather, there is no improper relativistic state of the system). But in general, when we bring in the local coupling of the network 'environment', the quantity m is no longer zero even though it remains minimally the scalar product of two vectors because these two vectors are no longer equivalent zero-vectors. The local equilibrium contains a tensor gradient on which time-reversal invariance has been broken, which means that the self-orthogonal components of each 2-manifold no longer have identical amplitudes (see *Section 2.5*) and operator *ordering* becomes significant, transforming from $\mathbf{R}_3$ to $\mathbf{R}_4$, and the system acquires improper relativistic states. The triplet group is no longer symmetrical in m under exchanges of all pairs of position coordinates. In other words the condition of preserved supersymmetry is itself unstable and is destroyed by local Lorentzian coupling, which is to describe the way that the supervening of accelerations due to 'other forces' on the idealised 'gravitational tensor' of flat, empty, homogeneous and isotropic space *is the same thing as* the emergence of non-zero improper mass. (Note: The extra degree of freedom rotates around the triplet as a vector product at each of three points of measurement, so that the *idealised* $\mathbf{R}_4$ equilibrium actually occurs in a tetrahedral sextet of vector products. See *Section 2.6*.)

54.) If time, mass-energy and gravitation are co-emergent in the ensemble then the work product, the mass *action*, will also be co-emergently continuous in $\mathbf{R}_4$, but discrete on a broken path in $\mathbf{C}_N$. Thus we conclude that continuous scale-specific gravitational action in $\mathbf{R}_4$ corresponds to a discrete scale-free null structure in $\mathbf{C}_N$ and has an extremal stationary value in $\mathbf{R}_4$ just as continuously-variable mass-energy in $\mathbf{R}_4$ has an extremal stationary value $\Delta m = 0$ corresponding to a discrete null element $|m|$ in $\mathbf{C}_N$. This absolute gravitational action $|g_a|$ must be zero in $\mathbf{C}_N$ but will have a minimum value $\Delta g_a = 0$ in $\mathbf{R}_4$. In fact we say that Δm and Δg_a are *the same tensor quantity*, not merely proportional quantities as in a field theory, and thus we avoid the issue of how the energy-momentum tensor couples to the metric tensor.

55.) In an effective-field representation in $\mathbf{R}_4$ we can use a perturbed state of the metric tensor to define local kinematical rest and so 'correct' the experimental masses of 'separate' free particles by reference to this independent spacetime gauge; but PM implies that this gauge will be inaccurate. There are no free mass monopoles in PM because the 'graviton' in this theory (see *Section 2.6*) is its own antiparticle, just as the photon is its own antiparticle. (Or to put this another way, to exclude free point-masses is to exclude gravitomagnetic monopoles.) We can only pick up a supersymmetric PM unit vector 'by its ends' as a complete parcel, as we have seen (*Section 2.1.* para.16), for a reason closely analogous to quark confinement, which is that to attempt to separate the two 'ends' would be to separate inertial mass m_i from its own reflected identity, gravitational mass m_g, or (in terms of $\mathbf{R}_4$) to separate the energy-momentum tensor from the metric tensor. Evidently the whole *meaning* of a constant positive local rest mass is just that the identical quantity m is measurable redundantly at both ends of any dyad when it is at *dynamical rest*, and this is equivalent to saying that the non-zero value of m measures an irreducible vector resultant in $\mathbf{C}_N$ which behaves as an inflationary vacuum energy or 'cosmological constant' in $\mathbf{R}_4$. In other words, can say that a 'dark energy' is *in* the rest mass.

56.) This is quite an interesting result. If two components of a vectorial dyad have jointly only half the rest mass energy as measured over a path in $\mathbf{C}_N$ that one would calculate by adding up scalar monads separately in $\mathbf{R}_4$ then this is equivalent to saying that there is a *notional* positive quantity $2m$ attached to every dyad in $\mathbf{R}_4$ which never appears in measurement because it is taken together with a hidden quantity $-m$. Or, the total apparent mass-energy M of pairs of cosmic masses calculated from summing particle numbers in $\mathbf{R}_4$ will need to be corrected by a *negative* energy quantity equal to $-1/2M$ to accurately model the gravitational dynamics. The result is that the simultaneous hypersurface of rest in $\mathbf{R}_4$ will not be quite where we think it ought to be.

57.) In the case of two locally 'free particles' we might expect to measure a total mass-energy of $2m$ when simultaneously at rest. But the local independence of m_{e1} from m_{e2} is exactly what makes simultaneity difficult to define as any definite state in $\mathbf{R}_4$. Weighing the two vertices 'simultaneously' requires us (conventionally speaking) to ensure that they are on the same spacelike hypersurface for the initial conditions of the measurement; yet the local affine structure of the manifold at any point in space is unknown *a priori* and could contain any arbitrary curvature, indexing not only a mass-energy density due to unknown charge distribution and other mechanical accelerations that we have to account for, but also an indeterminate curvature proportional to a 'cosmological' factor.[62] How are we to control the conditions of this experiment except by arbitrarily fixing a frame of reference for the purpose? This is of course what we must do, and so we stabilise things by an arrangement of other charges and masses; but this itself denies us the chance to disprove the PM contention that the whole supersymmetric unit carries mass m because we cannot separate one end from another so as to get a pair of monopoles, this being expressly the physical state that the foundational definition of PM excludes.

58.) By the same token, stating that each dyad has mass m does not mean that there is any sense in which each 'end' can be assigned a mass of $\frac{1}{2}m$. This is an inverted way of thinking, applying $\mathbf{R}_3$ as a constraint on $\mathbf{C}_N$. If we test this prohibition by comparing the weights of each end of the object consecutively in an identical gravitational gradient we will merely confirm that the identical result m holds for each end, surprising nobody of course. Conventionally, we say that by disturbing e_1

62 This reflects the existence of solutions of the GR field equations for spacetimes with zero mass-energy.

from 'rest' we elicit an inertial mass m_{e1} equal and opposite to an acceleration $\boldsymbol{a}$ which we suppose exerts no direct far action on m_{e2}, leaving e_2 at rest. Similarly if we apply $\boldsymbol{a}$ at m_{e2} we expect no far action to perturb e_1 simultaneously. Only field contact forces transmitted at the speed of light communicate on the line between them, producing a retarded local action. This appears to contradict the PM prohibition on separating one 'end' of the particle pair from another, and tells us that in general such separation is the rule. And indeed it is, inside $\mathbf{R}_3$. But any region $\mathbf{R}_3$ of the spacetime $\mathbf{R}_4$ represents a domain of broken supersymmetry that is a subset of $\mathbf{C}_N$. The measurable local *independence* of these two accelerations in $\mathbf{R}_3$, each equal and opposite to m_{e1} and m_{e2}, presupposes that we are able to *dependently* specify $\boldsymbol{a} = 0$ at e_1 and e_2 in the first place, i.e. to specify a *common* state of inertial rest in $\mathbf{R}_3$. This is problematical given that universal gravitation is universal acceleration, and this problem is exactly the reason that SR is generalised to GR, in order that any point state whatsoever may be independently regarded as 'at inertial rest' by the specification of a suitable gravitational field. Now we can see that the underlying motivation for this procedure is the redefinition of rest from a linear kinematical state function in $\mathbf{R}_3$ to a nonlinear dynamical state function in $\mathbf{R}_4$. But in GR $\mathbf{R}_4$ does not repair the broken supersymmetry. The significance of the underlying connectivity in the embedding set of $\mathbf{C}_N$ is that the continuous gravitational field in $\mathbf{R}_4$ corresponds to a quantised supersymmetric path function in $\mathbf{C}_N$. In other words, the GR-type continuum representation of the gravitational interaction defining the local condition of kinematical rest is only a mapping onto the state space of $\mathbf{R}_4$, an effective-field representation; but the gravitational *mechanism* (i.e., the theory of *quantum gravity*) lives in the path space of $\mathbf{C}_N$, not in the state space of $\mathbf{R}_4$, because the underlying 'absolute' state of rest is a dynamical path function not a kinematical state function.

59.) To recap: In general, δm and m_{mech} are identifiable with the free and internal energies. On any network path these quantities are properly cancelled against one another, reversible-closed-dyad by reversible-closed-dyad, like $m_i - m_g = 0$. Fermionic 'matter' and bosonic 'radiation' preserve a context-dependent supersymmetric ambiguity on such a path, which can be traced in terms of creation and annihilation of virtual photons and electrons. But these modes separate out in the irreversible 4-space representation, where $m_i - m_g \neq 0$. This is because what 4-space represents is a structure of intersecting paths. It is a state function corresponding to all possible different path functions in the state space defined in relation to some emergent kinematical zero-point of inertial 'rest'. In PM the path function is fundamental to the state function, and the breaks in the complex path are boundary conditions of self-orthogonal 2-manifold phases that interpenetrate linearly without regard to scale (the nonlocal subset of all pairs of electron states) instead of being nested volumetrically dependently on scale (the local subset of all individual electron states). In other words every set of states $\mathbf{R}_3$ lying on a 3-surface exists at the $N^{½}$ intersections of N members of a universal set $\mathbf{C}_N$ of complex 2-manifolds. Such a space has no true 'homogeneity scale'.

2.5 consistency with quantum mechanics

1.) The subtle ambiguity in the function of c (identified in *Section 2.3*) is reflected in the rapprochement between discreteness and continuity in PM. A change in norm of c means that the 4-momentum of a lightlike null vector is 'regauged', not *in* spacetime but in a PM state space that we have called $\mathbf{C}_N$. We can visualise $\mathbf{C}_N$ as a '*hyper*spacetime'. An electron recoil in $\mathbf{R}_3$ then represents a change in the hypermomentum of a photon track which, from a certain point of view, could be said to be continuous. This hyperspacetime is then the N-dimensional phase space of all PM N-vectors.

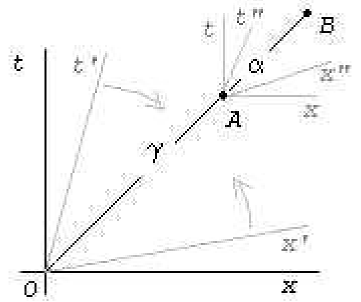
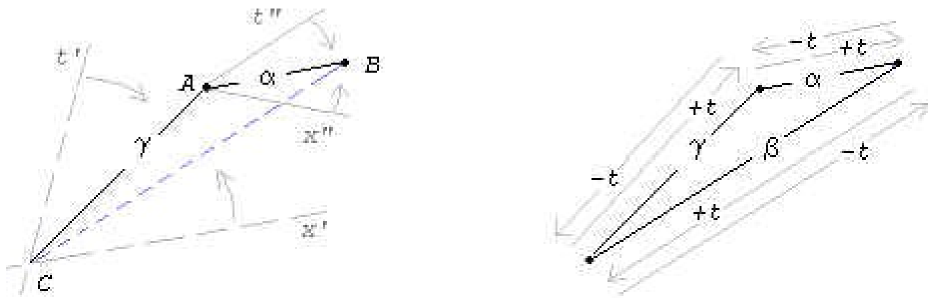


Fig.16

2.) Consider Fig.16. This can be looked at as an ordinary spacetime diagram which shows worldlines of γ and α as collinear lightlike vectors with congruent spacetime axes illustrating the case where c is constant on *OA* and *AB*. Also shown are arbitrary Minkowski-orthogonal axes for frames moving with different $<c$ velocities relative to both congruent rest frames x, t, to remind us that (as we saw earlier in relation to Fig.5, *Section 2.3*) a photon null vector collapses all axes like t', x' or t'', x'' onto itself and is considered to be *self*-orthogonal in its own frame. Accordingly one can say that c is *equivalent to all values of itself.*

3.) Adopting this hyperspace vantage, we can bring out the idea that light is always at rest with respect to all hypervector orientations of itself. Because each such hyperworld-line is self-orthogonal in spacetime coordinates we are free to break the lightlike line into arbitrary discrete elements of unit speed, rotated to produce a *non-differentiable acceleration* in the hyperspeed of light. To do this we have to superimpose different sets of coordinate axes in a very unorthodox way which would be untenable if light were a spherical wave radiated to infinity in continuous space (Fig.17*a*). But PM confinement ensures that path α remains a properly null zero vector consistently with all diagrams, because we allow photon α to define the set of electrons with which it is in a lightlike relation as those which confine it ('observe' it), in this case only *A* or *B* at either end of path α.



(a) *(b)*

Fig 17

4.) The break in the lightlike line automatically generates an elementary PM triad, where the spacetime origin is regauged at *A*, *B* and *C*. In Fig.17*b* the triad itself has now broken free from background coordinates and is self-referent. The interesting thing about this is the way that the pairs of antiparallel vectors each cancelled in the state of PM dynamical rest now find themselves transported nose to tail to give a counter-rotating 'circulation' of quantities like m_i and m_g. In the simplest case, that of a triad in isolated equilibrium, this has to be regarded as a *virtual* circulation where the quantities cancel to zero at all points of measurement.

5.) In isolated equilibrium the transformation of unit scale around the triad maps like 1:1:1 giving an equilateral figure with 120^o exterior 'scattering' angles, representing $\cos 30^o$ of negative work per vertex, summing to a total of $\cos 90^o$ around the triad. This means that (in this hyperspace representation) zero work of 'displacement' is done overall either in transporting a photon around the loop, beginning at any vertex, or in interchanging any pair of vertices (*virtual* electron positions). This gauge symmetry (see Fig.18) recapitulates the equilibrium of an isolated dyad inside a rudimentary triad that remains internally massless; in other words it is preserving a supersymmetric mode that realises *c* = *constant* in the form of unit time and unit scale. Here '*c* = *constant*' means '*c* = *unmeasurable*', in the sense that *c/c* = 1 is not contingent but is enforced for all possible measurements. The equi-angular triad enforces that the scaling algorithm for 'physical measurements' made by the system on itself recapitulates the degenerate identity of Fig.10, *Section 2.3*.

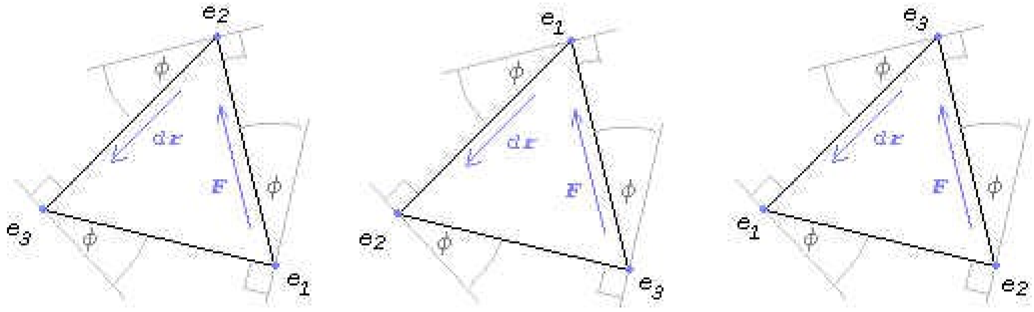
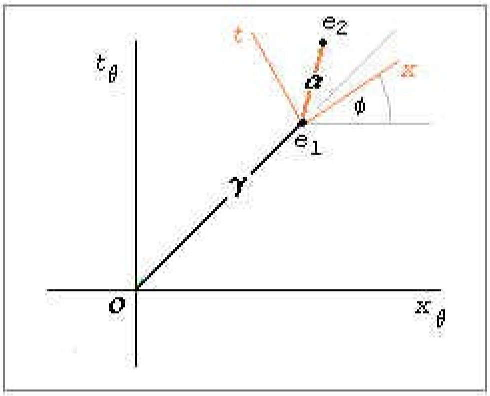


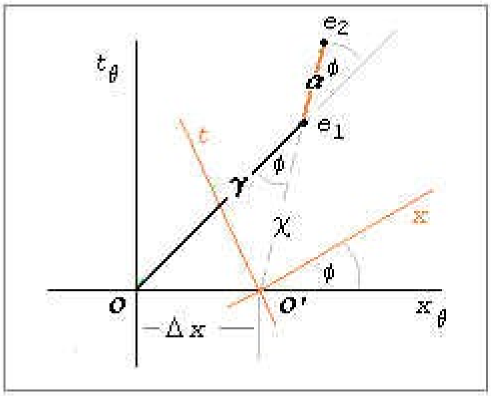
Fig.18. Triad symmetry. In one complete rotation each point exchanges labels with every other and each side is sequentially a force vector, ***F***, a displacement vector, ***dr***, and their scalar product ***F dr*** cos ϕ. If this vector direction represents $+t$ then the antiparallel direction (not shown) would be $-t$, and the cancellation on each side is reflected in the vanishing total (cos 90^o) of negative work done in taking a photon cursor around the loop and back (in *either* direction) or in transposing any pair of *virtual* electron positions.

But since isolation is impossible (this is a condition of the geometry of the complete graph in PM) this equilibrium configuration is in general not stable and 'decays' to another metrical state, in which state we say that the mass field is emergent from the radiation field. In this new asymmetrical state we can still trace a gauge loop of zero *summed* work around the triad from any vertex; but the work done is now not the same at each vertex and may indeed be positive on one of them. It is in the breaking of the idealised boson symmetry to this *dynamical* equilibrium that the fermion symmetry appears and the *virtual* electron positions in Fig.18 become real. (As we will see presently this can easily be visualised in terms of the transition from the zero-mass invariant spin-1 polarisation of a symmetrical photon ψ function to a triad of non-zero-mass spin-1/2 electron ψ functions.)

6.) Consider Fig.19 as a schematic hyperspacetime diagram for a c of varying norm, in which the tail of a lightlike line α is transported to the head of the lightlike line γ at a point which represents (say) an electron's scattering region between two photon paths. Since all frames are rest frames with respect to light the start of every photon hyperworld-line will bisect (Fig.19*a*) a pair of identically orthogonal hyperspacetime axes at $x = 0$, $t = 0$. As Fig.19*b* shows, the projection of α on x defines an origin O' which is simultaneous in the frame of O but hyperspacelike-displaced by the quantity Δx. The point O' does not appear to have a physical significance in the hyperframe of γ, or of α, but insofar as γ is a 'force' applied to e_1 in the rest frame of an electron hyperposition at O (ϕ being fairly directly proportional to photon momentum in the case of scattering, probabilistically so in the case of resonance radiation) it might be permissible to think of Δx as a virtual hyperspace displacement vector proportional to a quantity of work that would have to be done to cancel the change in the norm of photon speed through the 'hyperangle' ϕ from c_γ to c_α.



(a)



(b)

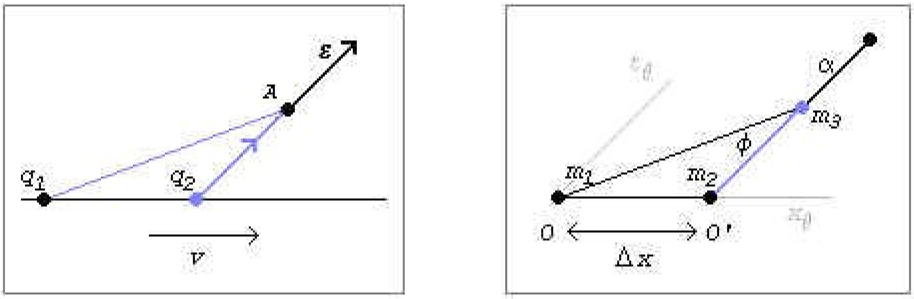
Fig.19. Schematic hyperspacetime diagram for multi-phase *c* of varying norm. The black axes t_θ and x_θ signify the hyperframe of an electron arbitrarily taken as 'at rest' at the origin *O* of photon hyperworld-line γ. Orange axes in *(b)* show the projection of the 'scattered' hyperworld-line α on x_θ. The displaced hyperposition *O'* is 'simultaneous with' *O*. Simultaneity is here defined by, and *only* by, a terminus of any photon hyperworld-line from *O* on which 4-space interval s^2 goes to zero, and so Δx not only *must* represent the distribution of an indeterminate position function for a 'pair of electrons', in fact it must represent this function for *all* such pairs. It is not a real distance on the hyperspacelike world-line of a particular electron, but rather the geometrical expression of PM *unit scale*, which is just the photon hyperworld-line connecting any electron position measurement with any other. We can see that what is significant about this geometry is not the absolute value of Δx (which varies arbitrarily with the hypercoordinates of e_1) but its *oscillation* on x_θ which is a periodic function of the rotation of the phasor α. The text explains why this becomes a complex function.

7.) The fact that ϕ has a non-zero value at all is just what determines that γ is no longer 'observable' at *O* ('because of' scattering or resonant re-radiation away from the lightlike hyperworld-line of *O* at e_1), but its actual (complex) value is proportional to a displaced hyperposition on x_θ which is 'seen' by an electron at e_2 as the hyperposition (simultaneous in the frame of *O*) from which α is emitted in the case that *c* is *globally single-valued*. If *c* is taken to be single-valued in a single-phase hyperspacetime continuum then e_2 could say that *O* is actually a *virtual* position of *O'* analogous to an optical refraction. In fact this is the point of view which, when 'scattering' by *mass* is considered in relativity, leads from the continuous flat manifold to the idea of a continuous curvature.

8.) This view corresponds to looking 'back in time' across a gradient of potential in a background space field that *causes* deflection of the lightlike line γ,α proportional to a local mass at e_1. But from our bird's-eye point of view, where *c* acquires arbitrary values in a multi-phase hyperspacetime, a more natural point of view is that *O'* is a virtual position of *O*, and instead of the curvature of a continuous medium due to a scalar mass at e_1 this gives us an inverted picture of gravitational deflection, where what we call the local mass of e_1 indexes a hyperspacetime 'dispersion' due to a discrete variation in the speed of light. We can call this a dispersion in the sense that c changes at e_1 simultaneously *for all space wavelengths* and then again at e_2 simultaneously for all space wavelengths, and so on. There is no dispersion between any pair of measurements, so it occurs not *in* spacetime but as a hyperspace phonon-like mode of the deflected lightlike string. The 'reason' for the deflection at e_1 is then no longer to be sought at e_1 itself, but in these phonon-like modes existing over larger hyperspacetime regions and quantising the 'refractive index' of the PM vacuum.

9.) How does this perspective help us? The hyperangle ϕ is proportional to a change in the hypermomentum of a photon, and more fundamentally that of any massless boson interaction - including therefore the gravitational interaction according to standard quantum treatments of GR. But how do we measure this? The 'actual (complex) value' of the hyperangle ϕ is an emergent representation of the self-consistent network of translational and rotational transformations, and therefore just an index of c itself; so its 'measurement' consists in the familiar relativistic process of establishing dynamical consistency in systems (the pre-metrical character of Δx is the whole point of the conception of PM unit scale, after all) and on that account our construction might be thought to be a redundant level of description. But from the point of view of PM's conception of a many-centred space the idea of a multi-phase c represents a generalisation of SR to a theory that 'includes gravity', *via* a type of complex vector construction that (as we will show in more detail in *Section 2.6*) evokes and extends the formalism of quantum mechanics. It allows us to say that a ratio of phases represented by the hyperangle ϕ acquires physical significance in terms of a projective superposition of 'spacetimes' that are not rotationally isotropic because their null-geodesic geometries differently, and privately, coordinatise the *complex* plane (every measurement triad defines a plane). This is the principle in terms of which we will be able to understand why mass is not a scalar particle property and does not 'live' at the point of its action (e_1), but rather is a nonlocal property of the emergent planar 'hyperfield' containing O, O' and e_1, and exists as a superposition of values of the unit vector Δx on the simultaneous hyperplane of O in Fig.19.*b*.

10.) We can visualise (Fig.20) how such a construction might be dual with a local field representation if we say that O is analogous to the so-called 'retarded position' of an electron e_0 at O' in respect of which the path α *would* remain a *massless null radial* component ($\boldsymbol{m}_{\mathrm{i}} - \boldsymbol{m}_{\mathrm{g}} = 0$) of the PM space field registered at O'. The hyperspace distance Δx travelling towards O can be regarded as the inverse of this mass/time-negating hyper-displacement. Therefore the complex hyperangle ϕ is proportional to a non-zero *positive* mass/time gradient on x_θ, equivalent to the *negative* change in hypermomentum of the photon α due to renormalising *c*, and associated with a hyperspace-like interval coupling *two position states of 'the same' electron*. This means that the shift Δ*x* 'takes place' on the hyperplane of *simultaneity* containing both *O* and *O'*, and evidently represents the PM projective geometric identity of point and line being expressed in the transition between a spacetime view and the view from the PM state space.



(a) *(b)*

Fig.20. Schematic illustration of analogy between *(a)* retarded spacetime position of charge *q* in electric field ε and *(b)* displaced simultaneous hyperpositions of mass *m* in PM space. In *(a)* the radial direction of **ε** at a time t_2 is associated with a moving charge at position q_2. But if a local action due to q_2 is limited by *c* then a test charge at *A* cannot instantaneously know the position of q_2. Therefore if the radius vector of the force $q\boldsymbol{\varepsilon}$ exerted at *A* is 'caused by' the moving point charge then it must be determined when the charge is still at q_1, the so-called 'retarded position' of q_2. In *(b)* a displacement $m_1 \leftrightarrow m_2$ on a hyperplane of simultaneity is a *doublet position state* in the projective geometry of PM hyperspace. Δ*x* represents the projection of *unit scale* as an arbitrary distance in any arbitrary spacetime frame, because in this hypergeometry of nonlocal couplings it is rotations, not scales, that have physical significance. The deflection proportional to Δ*x* is a hyperangle which is thus a constant of *all* local geometries (Lorentzian frames).

11.) PM's geometry ensures that although the transition Δ*x* is 'instantaneous' in the frame of *O* it does not violate momentum conservation, because it does not transport mass-energy. From the proper point of view of *O* (or of *O*') in which Δ*x* is radial, this is because of the cancellations $+t\ -t = 0$, $\boldsymbol{m}_i - \boldsymbol{m}_g = 0$ on x_θ. From the point of view of e_1 (or of any other *e* for which *OO*' is transverse) these cancellations are generally speaking *not* exact; but even so, Δ*x* is an interval whose 'centre of mass' neither spontaneously 'moves' in spacetime, nor remains fixed. There is simply no scalar position state in the 'middle' of our Einstein's-box that can be associated with a 'centre of mass', and what changes is a resultant of two opposed vectors, whilst a system of exhaustively interconnected self-orthogonal 1-dimensional spacetime 'boxes'[63] undergoes evolution by re-scaling in PM hyperspace so as to define 'rest' as the *dynamical equilibrium* preserved in, and between, all boxes. And thus, as suggested in *Section 2.2*. para.10, it becomes impossible to support a distinction

63 This means one *real* dimension; each 'box' is a doublet state that is actually complex 2-dimensional in terms of its advanced (real) and retarded (imaginary) wave representations.

between 'non-inertial' unit vector accelerations due to an applied force and intrinsic or 'inertial' accelerations where there is *no* applied force, which satisfies the **equivalence principle**.

12.) So a non-zero mass and time[64] on *OO'* emerges proportional to the changing complex hyperangle $Oe_1O' = \phi$ in a system where *c* becomes many-valued, and the properties of this geometry explain why these quantities are finite self-limiting. In general for most values of ϕ in real systems, $\boldsymbol{m}_i - \boldsymbol{m}_g \neq 0$ and $+t - t \neq 0$, because the triads are scalene like Oe_1O' and because each side has a multiple role in the PM geometry, not only as both force vector and as displacement vector (Fig.14) but also as a resultant. Thus the 'length' of each side (its projection of unit scale Δx) is a *superposition* of three interdependent vector magnitudes each specifiable in three different ways, whose various combinations generate different internal angles. In these cases it is clear that the scalar product for work done on any side depends sensitively on the *order* in which the operator values of the remaining two sides are taken, and that the matrix of cyclic permutations of these logical orderings produces a periodic oscillation which is equivalent both to the permutations of position and momentum coordinates in the Heisenberg picture and to the differentiation of the wave function with respect to time in the Schrodinger picture. This results in the *anti*symmetric wave function where $pq \neq qp$ for *most* combinations of values of ϕ, *p* and *q* (force ≈ momentum, *p*; displacement ≈ position, *q*).

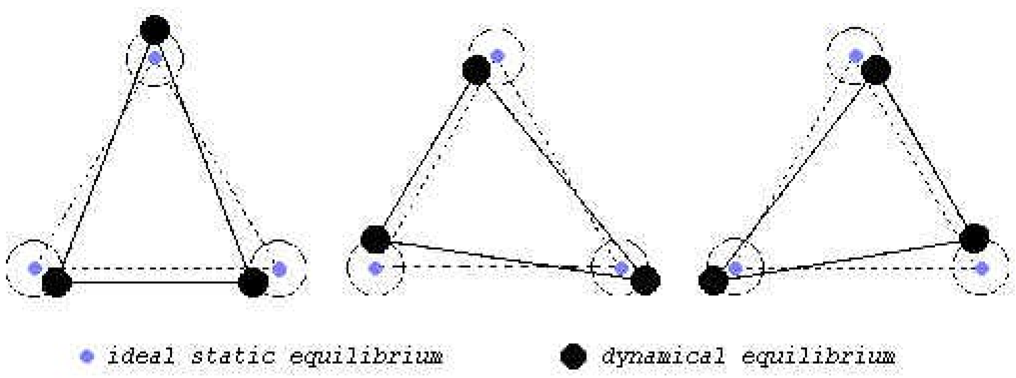


Fig.21. One cycle of 'operator orderings' in an equilibrating antisymmetric triad. 6 different arrangements of spin labels are possible for each of the 3 isosceles configurations. (Open circles are imaginary libration 'orbits' on which the vertices lie, but are not traced continuously by the vertices.)

64 *Important reminder*: PM antiparallel-vector dyads mustn't be seen as simplified miniature facsimiles of the large scale cosmos, containing properties like 'time', 'mass' etc., neither individually nor even in systems that are numerically quite large compared to the schematic triads considered here. What we call a 'mass vector' or an 'advanced wave in the negative time direction' are concepts derived from theories of complicated systems. But the system properties we label as 'time' and 'gravity' in the ensemble arise from a rudimentary dyadic PM symmetry which is itself a dimensionless quality.

13.) If we assume a static condition without time evolution for simplicity, what can we say about the system for different values of ϕ? Firstly, given $p = q$, when $\phi = 60^{o}$ we have the unique case of an equilateral triangle. This recapitulates the equilibrium condition illustrated in Fig.18, where the operator ordering is immaterial and therefore all three points of measurement lead to *symmetric* ψ functions. This is of course not generally a configuration available to fermion systems since the emergence of 'points of measurement' is cognate with transition to a dynamical equilibrium of *antisymmetric* ψ functions. If we characterise the static equilateral triangle as a continuous photon loop (equivalent to a counter-rotating anti-photon loop) containing virtual electron/positron position states, then we can see that comparison of the invariant photon polarisation vector with itself is always consistent because it rotates around the direction of the 'world line' to return with the same phase. On the other hand, the transition to a fermion representation destroys spin consistency around the loop because the spin components lie on directions Minkwoski-orthogonal to the world line, meaning that only two out of three possible comparisons of fermion spins can form opposite Pauli pairs with otherwise identical quantum numbers that are antisymmetrical under position exchange. One possible pairing must always be *symmetrical* in the two coordinates. This leads to an inevitable dynamical instability in the position states, which we interpret in terms of the cyclic permutation of operator orderings on a 'rotating' sequence of isosceles triangles where in general $pq \neq qp$ for two out of three operations (opposite orderings leading to different fermion vectors) but where $pq = qp$ for the third. One possible cycle of dynamical equilibrium configurations is illustrated in Fig.21, where the two-fold degeneracy of the triangular symmetry group is that of the '*E* representation' in group theory. (We can picture the *complex dynamical* equilibrium at each vertex - in terms of its *real kinematic* projection - as somewhat analogous to the libration of a mass around a stable Lagrange point in a gravitational three-body problem where the 'dominant' masses and the stable smaller mass at the 'L4' or 'L5' point are continually changing roles - i.e., mass is an emergent system property that can be considered to rotate around the triad. At every third 'instant' there is one mass-vector pairing which has a smaller magnitude in equilibrium with two equal vectors of larger magnitude. The mass of the system is a nonlocal property that cannot be rigorously isolated inside it.)

14.) Now we need to identify the special case of right-angled isosceles triangles. One way of looking at this is in terms of the schematic mapping of Fig.22. It is easy to see that as the angle ϕ increases, and as χ becomes parallel to x_θ, Δx grows towards infinity (or unit scale realises the 'line at infinity' in the projective PM geometry for the point e_1; see also Fig.3, *Section 2.1.*). But we find that there is a sudden scale-free phase change whilst Δx is still finite, a discontinuity at $\phi = 90^{o}$

where the action (rate of work done, proportional to cosϕ) appears to go to a minimum on both γ and χ, regardless of the order in which the operators corresponding to force and displacement vectors are taken. If (continuing to assume an ideally static case for the moment) we take this minimum to be exactly zero for exactly cos90°, then this is obviously equivalent to saying that the radial force on both γ and χ vanishes, a condition which is mirrored in Fig.22 by the condition that $+\Delta x=-\Delta t$, or *OO'* becomes self-orthogonal and causally null *for the purposes of measurements at* e_1. So we deduce that a normal antisymmetric commutation relation for the transverse displacement vector must change at a phase of ϕ = 90° to a symmetric relation, *uniquely for 'observers' like* e_1, appropriate to a pseudo-boson where a 'pair of electrons' exhibits long-range correlation of the EPR type.

15.) The vital qualifier here is 'uniquely' because it reassures us that this pseudo-bosonic relation is not accessible singly at *O* or singly at *O'*, but only *doubly* at e_1. This explains the preservation of SR locality inside a physics of nonlocal correlations: *It is necessary that a correlated pair of states be cross-correlated through a third common state*. That is to say, the correlations in Aspect-type experiments are between states that in terms of the geometry of Fig.22 are carefully prepared at e_1 (for example a proton spin singlet, or a photon pair from an annihilation event) and transported to *O* and *O'*. The no-signalling condition therefore holds in PM between any given pair of vertices.

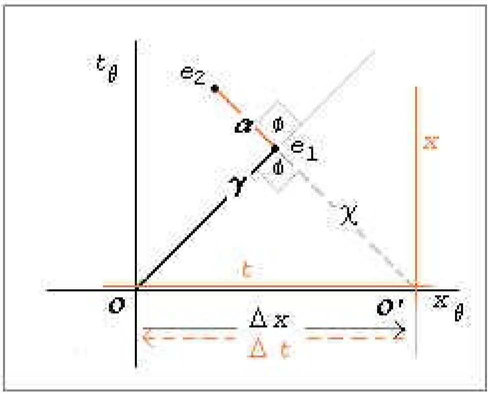


Fig.22

16.) As ϕ reaches 90° the *t* and *x* axes of the simultaneous hyperframe containing *O* and *O'* are collapsed onto one another, becoming antiparallel and interchangeable, so that $+\Delta x_\theta-\Delta t_\theta \equiv -\Delta x_\theta+\Delta t_\theta = 0$. This implies that the null mass-vector $\boldsymbol{m}_i - \boldsymbol{m}_g = 0$, which in general characterises only the (proper) *photon* state of the self-orthogonal radial vector *OO'*, will be recovered also as an (improper) *electron* state for ϕ = 90° measured at e_1. Therefore we expect (again, in this time-free idealisation) that the scale-free onset of long-range EPR correlations on *OO'* will accompany the vanishing of force due to mass between *O* and *O'* as measured at e_1.

17.) Now remember that *OO'* stands for *all* self-orthogonal spacetime radius vectors at *O* (or *O'*) and represents a constant of *phase* in PM space for e_1, not a constant of scale. This result is actually independent of the mapping convention in Fig.22, as we have seen; indeed the fact that a diagram like Fig.22 can be drawn for any set of lightlike-related points like e_1 and e_2 tells us that for *all* points like *e*, taken as the origin of a unique set of self-orthogonal null lightlike radius vectors like γ, *all* real transformations (Δx) of *all* transverse intervals between radii γ_1 and γ_2 'seen' by *e* represent, independently of scale, a phase change which we are permitted to say corresponds to a *hyper*angle of 90° (to be formally represented as a rotation by *i* in the complex plane of the angle *OeO'*, as already indicated) where termini like *O* and *O*' become hyperspacelike-simultaneous and recover an equivalence expressed (in the form of a photon/anti-photon) as a degenerate identity at 0/180°. This, we suggest, is why the commutation rule for position states q_1 and q_2 of any electrons e_1, e_2 whose hyperphase subtense ϕ = 90° changes from a wave function with an antisymmetric solution

$$\psi_{e'}(q_1)\psi_{e''}(q_2) - \psi_{e'}(q_2)\psi_{e''}(q_1) \tag{28}$$

to a symmetric solution

$$\psi_{e'}(q_1)\psi_{e''}(q_2) + \psi_{e'}(q_2)\psi_{e''}(q_1) \tag{29}$$

which describes *opposite spin pairings* of electrons with zero relative angular momentum generating the bosonic integer-spin phase of the supersymmetric PM doublet.

18.) So what is the reason why, in the real ensemble, spacetime projections of pairs like e_1 and e_2 do not - *in general* - correspond to this hyperangle ϕ = 90°? To answer this we can interpret the phase transition in the following way: The hyperplane of simultaneity containing all transverse intervals obeying expression.22 can be described as a critical-point phenomenon, a transition occurring in an *n*-dimensional system that behaves with *reduced dimensionality* where the correlation length becomes equal to the confinement 'distance'. Consider an analogy with ferromagnetism: In a 3-space system of spins confined between parallel planes, the high temperature correlation length can be small compared to the distance between the planes and the system behaves 3-dimensionally. If the temperature is lowered through a critical region until the spin correlation length exceeds the distance between the planes then the system starts to behave 2-dimensionally. In our case the confinement distance and the correlation length are both just *unit scale* at a critical transition defined by the complex phase angle $\phi = 90^o$ measured at e_1, representing a rotation from the pure real plane to the pure imaginary plane which is naturally a time-reversal. In other words, under a series of multiplications by i, the n-dimensional system behaves like a lattice of correlated spin pairs confined on the two dimensions of a hyperplane. This critical-phase transition occurs for any e. But - and this is the important point for our many-centred PM formalism - the meaning of the fact that ϕ is complex is that $e_{1,} e_{2,} e_3 . . . e_n$ cannot live on the *same* hyperplane.

19.) We can see this as another statement of the PM exclusion principle. There is no single *global* operation applicable to every e; but instead, in going from e_1 to e_2 there is one well-defined rotation by ϕ which takes us from a complex plane C_1 with imaginary and real axes i_1 and r_1 to another, C_2, orthogonal to it in the plane of i_1, in which i_1 and r_1 (representing x and t on C_1) are collapsed self-orthogonally on either i_2 or r_2. Each move from one point of measurement to another (one vertex to another) incurs this rotation through ϕ and thereby exchanges one plane of complex coordinates for another orthogonal plane, in which the self-orthogonally collapsed axes of the former may now be specified either as a pure real or as a pure imaginary 1-axis for complex arguments in the new plane (see Fig.23). Thus every unit scale Δx has both real and imaginary roles in *n-complex dimensional* PM space, which is of indeterminate dimension yet is always (hyper)complex planar for any measurement. We can start to see here the way in which the factor $\phi = i$ behaves as a renormalising factor for c, so that moving from vertex to vertex through multiple broken phases of the lightlike line takes us from one complex plane to another in a heirarchy of operations that generates a 'hypercomplex' matrix of nested quaternions, octonions and sedenions.

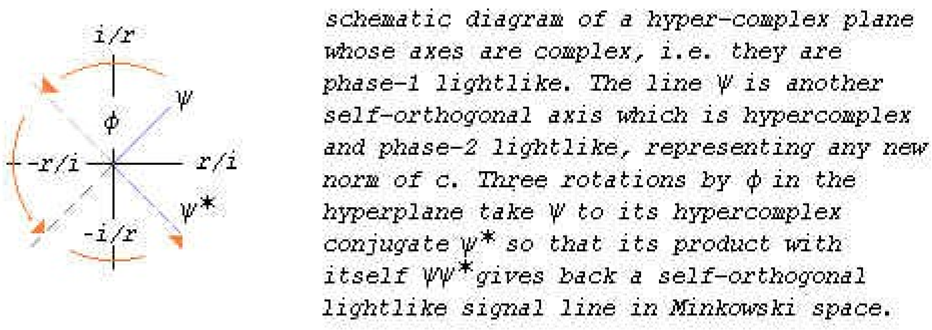


Fig.23. A rotation ϕ taking r to i in the hyper-complex plane renorms c for a measurement at a new vertex. Three such rotations close a triad of unit vectors in equilibrium.

20.) Returning to the plane of Fig.22, when we bring in the periodicity we are saying that the *rate of change* of the projection of unit scale Δx is proportional to a rate of work done equally on radial vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\chi}$, which goes to a minimum when the value of ϕ *oscillates around a stationary value* of 90°. This zero-point of work is a constant of all projections of this shift of complex phase, so it is independent of metrical distance. It is a lower bound on the work of transforming unit scale in n dimensions, superimposed on or ulterior to the relativistic composition of velocities taking place among 4-vectors in Minkowski spacetime, i.e. a constant underneath these local-real displacements which cannot itself appear as a dynamical *variable* of the Lorentz symmetry group. Nevertheless work is a dynamical quantity, which has the same dimensions (energy x time) as **action**. In other words the quantum condition originates in an emergent complex periodicity of the vector geometry of PM hyperspace.

21.) So we trace the quantum condition in general to the breaking of the direction of the hyper-lightlike line in PM space and infer that the condition $\phi = 90^{\circ}$ defines a *plane of constant complex phase* on which the rate of *change* of work goes to zero, meaning that time goes to *unit time*, because ϕ is renormative indiscriminately, and there is no clock. The projection on spacetime of the quantum of complex work is a transformation factor i applied to the rate of change of an action that oscillates around a common zero-point condition which we guess to be equal to the Planck constant, h. This oscillation comes from a cyclic permutation of operator orderings, and in a non-equilibrium system with many phases it does generate a clock, so that relativistic spacetime emerges in the renormative calibration of all clocks for the condition $c = const$. But as we begin the primitive stages of assembling such a clock we encounter the lowest determinate energy state in an equilibrium triad where the *rate of change of work* sums to zero around a neutral gauge loop (Fig.18). This is a minimum condition equal to a zero rate of change of energy, but not an absolute zero condition for the triad because measurement is a register, not of E itself, but rather of ΔE. The physical significance of E here is only as the vacuum energy, a zero-point of an isolated equilibrium gauge for a 3-dimensional flat space. It is flat in the same sense that a surface is flat when transport of a parallel-displaced vector around a closed loop returns it to the start with no change of phase. The hyperplanes at $e_{1,}$ e_2 and e_3 are identically interchangeable. In more complicated asymmetrical systems, actual systems where real clock rates emerge, the space will (in general) no longer be flat in this sense because (in general) the hyperplanes e_{n} are not indiscernibly identical.

22.) To expand on the gauge symmetry: In PM the concept of the rate of change of phase of '*a* photon' recedes to an abstraction and has no real physical meaning. On the PM network the physical quantity is a rate of change of phase determined over some *sequence* of photon states or pairs of measurements, *A*, *B*, *C* etc. We can compare photon phase at *A* with phase at *B*; but we *cannot* compare a photon with a future state of itself at 'the same place' because a photon can never be brought to rest for any observer. Hence, although we believe we can compare the phase of an electron with itself at the 'same place' - i.e., compare phases at *A* at different times[65] - we can only ever compare photon phase at the same place *A* and at different times by comparing the phases of *two* photons. If we wish we can describe the situation where a sequence of such pairs forms a *closed loop* that returns phase to its original value as an extremum where the rate of change of photon phase goes to *zero.*

23.) In an abstract sense, this is what happens because of complex time-reversal symmetry inside a single pair of vertices; but in an observable sense, in terms of local-real Lorentzian symmetry, it only happens on a closed loop of at least three such pairs - a triad. Either way the phase relation is anchored at a specified point in space (a vertex) because the photon only exists at emission or absorption. One way of saying this is that a photon phase 'shift' only occurs as a mutually-cancelling 'comparison' of phases of creation and annihilation, which are directly the Hamiltonian operators of one another, where the phase shift θ – θ' around such a gauge loop is *self-consistently and indiscernably* zero (or unity). But a finite speed of light means that θ :θ' involves an interval of time for all possible observers; or from the other point of view, the fact that θ – θ' ≠ 0 imposes the condition $c \neq \infty$. By the same token the fact that θ – θ' = 1 is only 'measurable' between *different photon states* around a real external loop that introduces a non-zero interval of time, and not around the null complex internal loop of time-reflection that brings the *same* photon state back to itself in zero time, is equivalent to saying that $\Delta c \neq 0$ - the *difference* in *c* is not zero - or the hyperspacetime momentum of the photon world-line breaks at some points.

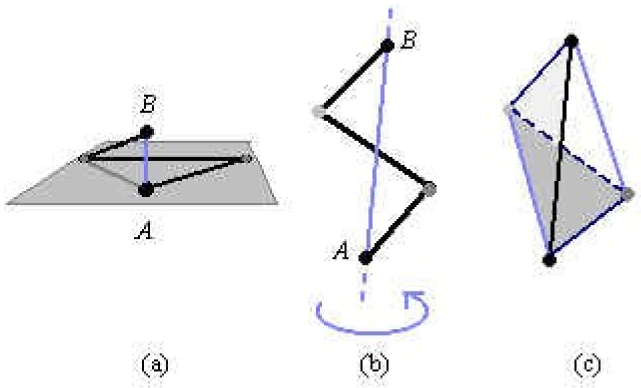


Fig.24

65 It is actually more complicated because 'observing an electron' is interacting with a photon, so all observations in QED are ultimately 'observations of photons'.

24.) To understand this, consider that the PM exclusion principle is what turns null photon time into timelike displacement (see *Section 2.3*). This can be illustrated by a recursive 'spiral' loop which returns photon phase *A* not to itself but to a partner state *B* displaced by at least one turn of the spiral (Fig.24a, 24b). This spiral minimally traces a tetrahedron[66] in PM vector space (Fig.24c), or a 4-dimensional sub-manifold of hyperspacetime, where the 'pitch' of the spiral, a displacement 'in time', is the resultant of a cycle of vector operations in the state space which is equal to some multiple of $\hbar / i$ and transforms from one triadic hyperplane to another. This cycle of operations applies to each of the dyadic elements in every spiral, and the fact that it always produces another dyad expresses the PM exclusion principle, which we can now see is indeed, as already observed, equivalent to the SR condition $c \neq \infty$, but requires *also* the PM condition $\Delta c \neq 0$ to avoid the degeneracy of singular states to which SR would lead if all hyperplanes were transformable onto one another in a common global frame of reference. In this sense the multiple phases of *c* are interpretable as *n* exclusive hyperspacetime quantum numbers of the system identifying *n* unique states with restricted occupancy.

25.) Where a minimal triadic loop of half-wave fundamental string modes gives a half-integral phase shift and does not simply reinforce, there is an odd number of half-wave modes in the loop and we say there must be 'an electron' in the loop. And the fact that photon creation and annihilation phases always involve an interval of time (finite *c*) means that there are always electrons in the photon loop, just as the fact that intervals of measurement are distinguishable means that there are always photons in the electron loop. The two modes cannot be separated from their joint supersymmetric *anyon* representation, meaning that what underlies the PM gauge symmetry of 'charge' is that there *are no open trajectories*. All paths form parts of closed gauge loops. In ideal equilibrium this gauge would 'crystallise' in the form of a neutral, massless coherent state on a single supersymmetric loop; in actual far-from-equilibrium systems the measured photon phase shift will be an incoherent superposition of different values each corresponding to a different electron loop.[67]

26.) In terms of PM's reinterpretation of the transactional 'absorber theory' formulation of quantum mechanics such an operator loop is readily visualised. Fig.25(a) shows the conventional transactional spacetime picture in which the 'absorber' oscillator, a particle e_2, is stimulated by the

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66 The tetrahedron of vector products is not possible in $\mathbf{R}_3$ because each of the three products generated from any triad must be orthogonal to both of its components and so the products only meet at infinity. It is possible in $\mathbf{C}_N$ because *all* hypervectors are 'orthogonal' in the sense necessary to close all hypervector products into a finite complete graph.

67 Spin 1 and spin 1/2 are interpretable as 'visible' modes of a broken *superspin*, a supersymmetry whose indefinite numbers of phonon-like modes, corresponding to all possible spin fractions, exist over the loops that would be traced by all possible closed paths through *all vertices* of the entire PM graph.

retarded wave arriving from the 'emitter' oscillator e_1 to emit a retarded wave which is exactly 180° out of phase. The two waves cancel in the positive time direction. But the accompanying advanced wave travels in the negative time direction, reinforcing the amplitude in the region e_1 - e_2. In the schematic picture in Fig. 25(b) the background coordinate space is lost and the system is closed on itself. As shown, the primitive equilibrium of Fig.18 is subverted due to selecting e_1 - e_2 as emitter and absorber, and what was there a zero-point symmetric wave function on the whole triad here collapses to a positive-amplitude antisymmetric wave function for the pair of charges e_1 and e_2. But in the absence of definite constraints this particular transition is an arbitrary choice. Why e_1 - e_2? Why not rather e_2 - e_3 ?

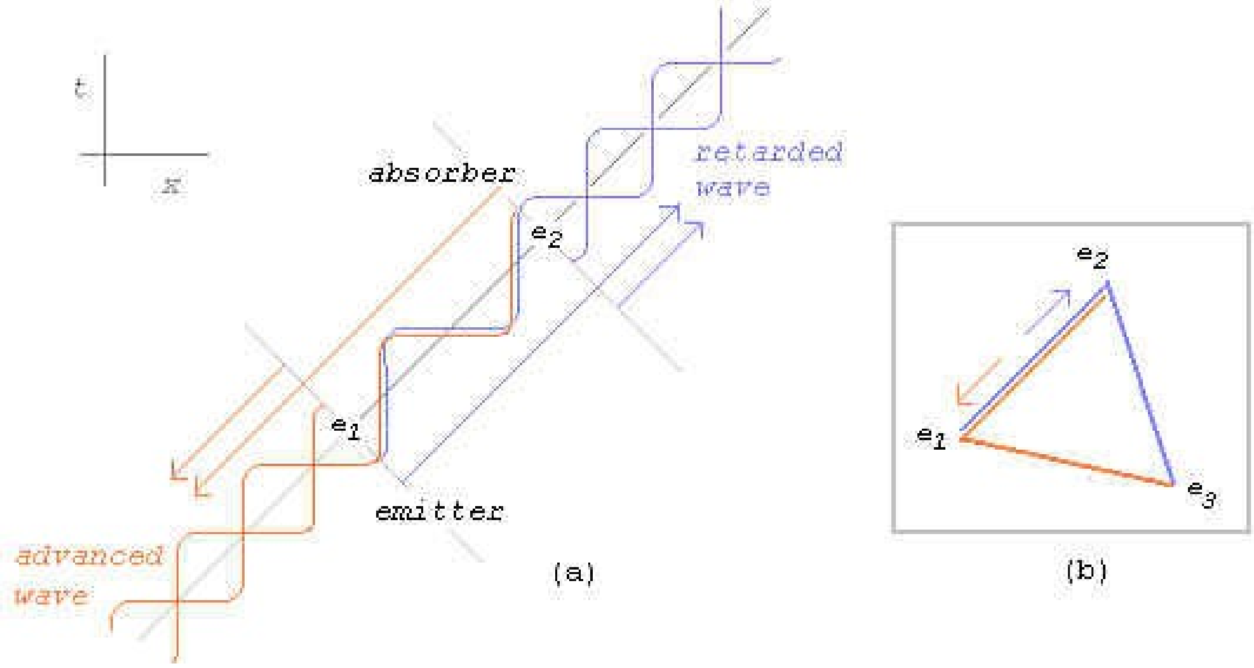

Fig.25. An adaptation of the 'absorber theory' model for a multi-phase *c* network. On the left (a), a Minkowski diagram shows mutually-cancelling waves propagating collinearly in a lightlike direction with zero-amplitude to past and future infinity. In PM (b) the lightlike directions are no longer collinear but are self-orthogonal spacetimes. Past and future do not exist in this diagram and time-symmetric advanced and retarded waves complete a hyperspace triad at e_3.

27.) We have to assume that a triad in equilibrium retains ideal causal symmetry, and there is no reason not to make the same reduction in favour of e_2 - e_3 and e_3 - e_1. Indeed, without doing this there is no way of understanding how e_1 can emit in the first place without violating energy conservation. When the causal cycle is completed, a reinforced wave of half-advanced and half-retarded amplitudes on e_3 - e_1 is revealed as the origin of the quantum operator responsible for the 'spontaneous radiation' at e_1 with which we began, and so on around the loop. As we saw (Fig.17*b*) this triad of interdependent oscillating states can be expressed in terms of counter-circulating positive and negative time vectors, and the two wave function solutions represent positive and negative energies; so we have preserved an idealised zero-point equilibrium for a triad of three electrons. By ‘keeping the three plates spinning’ simultaneously, so to speak, we are preventing the system collapsing to a definite positive-energy antisymmetric state, and our ‘three fermions’ in equilibrium are behaving like a Bose condensate.

28.) It is easy to see that the spontaneous radiation connected with an ‘infinite’ number of virtual photon basis states in the Dirac formalism is here connected with the time-reversal non-invariance of emergent local $\mathbf{R}_3$ domains under the super-rule that all paths are *closed* paths in (finite) $\mathbf{C}_N$. Consider a linear range of points (measurements)

$$A \rightarrow B \rightarrow C \rightarrow D$$

lightlike separated in a Minkowski spacetime. The absolute square $\psi\psi^*$ for the photon probability amplitude is always real at the point of measurement (say B) but this amplitude, which as a probability must be projected in the ‘future’ direction from point B, can be regarded as the ‘output’ of a zero-amplitude complex superposition occurring in the ‘past’ direction from point B. When the real amplitude $\psi\psi^*$ reaches point C it vanishes again into a complex superposition projected in the ‘future’ direction beyond C. But the primitive symmetry of BC contains no sense of a distinction between past and future and so this entire sequence is perfectly reversible, with the amplitude $\psi\psi^*$ being equivalently interpretable as the past-directed ‘output’ from a future state beyond C vanishing at B.

$$A \leftarrow B \leftarrow C \leftarrow D$$

The flat-line complex regions ‘before’ and ‘after’ BC represent *zero*-probability amplitudes for emission in both $-t$ advanced and $+t$ retarded directions to infinity, which in PM represents the boundary conditions for an isolated dyad.

$$A \leftarrow B \leftrightarrow C \rightarrow D$$

This is fine, but then we would have to ask how zero amplitude waves going to infinity on the past and future sections of the line can give rise to the non-zero probability of ‘spontaneous radiation’ inside BC.

29.) In quantum field theory, of course, the zero amplitude waves do not represent a state of minimum energy. Indeed the vacuum fluctuations *outside* the region *BC* are *greater* than those inside, due to exclusion of certain wavelengths of the zero-point energy. But it is an unsatisfactory feature of this construction that the zero-point energy is not well-defined. It explains in principle why spontaneous radiation does occur, but it is unsatisfactory that the zero-point energy outside *BC* is *infinitely* undefined. The usual absorber construction in the spacetime representation, indeed, seems to require that the vacuum potential well containing *BC* be infinitely deep, else how is it possible to be sure that amplitudes for spontaneous emission to from *B* or *C* to past and future infinity outside of *BC* are zero? In QFT one has to be optimistic that some future quantum-cosmological theory might absorb the infinite discrepancy into a finite natural relation - as discussed in *Section 2.4.*, this is connected with the ill-defined status of mass and charge renormalisation in quantum mechanics. In PM on the other hand we start with basic geometric definitions which disallow states that are open to infinity.

30.) PM allows nature to resolve these connected 'problems' for us because nature is conceived on the deepest level as a system of dyadic self-interactions, discrete path functions supplanting the continuum state functions of $\mathbf{R}_3$ representations. Interactions or measurements are interpreted as a breaking of the direction of the lightlike (hyper)line at *B* and at *C* along with a breaking of the primitive symmetry inside *BC*. Breaking the lightlike line destroys the exterior zero-probability amplitudes in both *-t* advanced and *+t* retarded directions by replacing them with non-zero amplitudes on finite dyads like *AB* and *CD*. In other words, it is only because *AB, BC* and *CD* do *not* lie on the same lightlike Minkowski null vector to infinity that the verification by measurement of the full superspin symmetry-breaking transaction in *BC* occurs, inside *AB* and/or *CD*. And it is this process of transactional verification by external measurement that generates a non-zero amplitude for 'spontaneous radiation' inside *BC*. (In a sense radiation is 'spontaneous' to the extent that the advanced-wave solutions for a quantum system are excluded for the special purpose of making theoretical models that simulate the time-dependent evolution of human ignorance. Of course, excluding the advanced-wave solutions is not a free choice for us; nevertheless nature does not 'care' about the local-real predictiveness that we value in these models.)

2.6 quantum gravitational mechanics

1.) The complete PM space of N unit objects is from one point of view a space of N mutually orthogonal 2-complex-dimensional subspaces,[68] also visualisable as a continuous self-intersecting curve, or as a complete graph exhaustively interconnecting $N^{1/2}$ network vertices or nodes. It is to be the complete state space of physical interactions, which are changes of state of these radically-orthogonal unit objects occurring as the curve is self-similarly ramified in self-interaction. Because it exhausts all of the space it generates, then by definition it is always a space-filling curve (and has, in this general way, the topology of any closed, finite but unbounded space); but it is not a space-filling curve in the sense of a Peano or Hilbert curve which is considered to fill a *background* space with infinitesimal copies of itself after infinite iterations: The PM exclusion principle coming from our basic geometrical definitions (Section 2.1) does not permit graph edges to go improperly to metrical zero, which is the same as saying that it is not physically meaningful (even if it may be theoretically convenient for certain purposes) to define a *real* (Lorentzian) continuum scalar background external to the graph. All position/momentum states on the graph are to be functions of other position momentum states on the graph.

2.) However for certain purposes a fractal tiling of all self-similar PM triads can be described as tending to a *hypersurface* of limiting topological dimension 2 which exists where the correlation length in $\mathbf{C}_N$ is universally equal to the 'lattice' separation in **R**.. We have identified this phase (*Section 2.5*) as analogous to low-temperature reduced dimensionality in quantum films, so that a physics of definite real quantities is emergent at this critical point of a system with reduced fractal dimension. We will call this complex fractal-2 hyperplane the $\mathbf{F_2}$ hyperplane.[69] It exists at what we will call *the critical point of anyon supersymmetry*, and it identifies a (theoretical) limit in which values of m_i - m_g can be said to go to a zero-point identity on every self-orthogonal 2-manifold of

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68 The meaning of complex dimensionality that emerges from PM is that the breaking of a primitive *reversible* symmetry on the theory's unit object (specified by an exclusionary principle in our foundational definitions - see *Section 2.1*) is what underlies both the spacetime structure and the complex vector-space representation of wave functions in QM. In abstract isolation the unit object rotates onto itself in a fully congruent transformation; but in the ensemble (in the full PM space structure corresponding to the Lorenztian 4-manifold of relativistic mechanics which we call $\mathbf{R_4}$) the primitive unit symmetry is broken to a dual-viewed *binit* symmetry of twinned unit objects whose rotational space self-transformations are now no longer fully congruent. Neither twinned unit can rotate onto itself identically, but they remain coupled as antiparallel components of a complex *self*-orthogonal object and may be made *mutually* congruent through what is now (emergently) a *time* rotation. The real half of the new symmetry space contains 1(real)D *irreversible* (+*t,* time-dependent) units in SR vector space; the Minkowski-orthogonal half, 1(imaginary)D irreversible -*t* units. (Labelling of real and imaginary dimensions on the subspace is arbitrary in terms of the abstracted unit object, of course. The time signs that differentiate antiparallel vectors in the decoherent ensemble reduce merely to inter-rotatable reciprocal views of the same interval.). In the photon/antiphoton representation of SR these Minkowski self-orthogonal components are properly real-null; but the complete local ensemble symmetry needs also a fermionic representation involving *improper* (transverse) real rotations in which the interval is non-null. The underlying broken symmetry is carried into this complete $\mathbf{R_4}$ as an embedded transformation subspace, the space of *intrinsic spin* transformations.

69 Each triad of nodes can be thought of as a different planar instantiation of the underlying *flatness* of $\mathbf{F_2}$ inside $\mathbf{R_4}$.

unit scale, coordinatised by a pseudo-scalar hyperplanar lattice of mutually Minkowski-orthogonal zero-point mass vectors. The idea is that if we reify the complex vector space(s) of QM as a transform of this hyperplane it will produce the primitive unit vector state space of PM quantum gravitation.

3.) The principle of the latter is that in the symmetry-breaking operation that produces *improper* (local, transverse) vector quantities from the $\mathbf{F_2}$ hyperplane, the zero-point supersymmetry is preserved on all *proper* (nonlocal, radial) connections. The result of preserving absolute (exclusionary) unit scale on a discretely vectorial network inside a relativistic structure is equivalent to a vacuum-driven *inflation* underlying emergent real scale on the signal line between every network node. The vector resultant at each node is a constraint equivalent to an *inertial mass*. Thus PM would have an effective tensor field representation in a state space of 4 dimensions which can be described as a *vector inflation* theory where an imperfectly isotropic cosmic smoothing is the inverse of a local anisotropy called gravitation. We conjecture that such a natural relation removes (in principle) the need for arbitrary fine-tuning of multiple scalar constants.

4.) As opposed to a local flat sheet where *all* position states would be occupiable, we can think of $\mathbf{F_2}$ as a *non*local flat sheet where *no* position states are occupiable.[70] That is, it contains no ‘filled’ particle states, or contains no points of self-interactive measurement; and insofar as it is empty (and *only* insofar as it is empty) it is equivalent to a differentiable 2-manifold. The filled states are to be coemergent with the deformation of the nonlocal lattice, producing a ‘curvature’ of the nonlocal plane to a *non-differentiable* N-complex-dimensional hypersurface associated with the action of mass, in the following way:

5.) As $\mathbf{C}_N$ attempts to generate a real geometry of flat $\mathbf{R}_3$ space out of the superposition of $N^{1/2}$ different ‘views’ of itself at $N^{1/2}$ nodes of measurement, it finds that the network is unable to preserve equipartition of scale (i.e., equipartite scale = absolute scale); instead, the *self-consistency*

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70 The transformation of this notional surface to $\mathbf{R}_4$ preserves the nonlocality on a radial phase of the PM graph, but the full actuality of $\mathbf{R}_4$ depends on the processes of complex measurement (=self-interaction) which involve intrinsically the network’s imaginary ($-t$) states in the physical meaning of its real ($+t$) states. In other words an *individual* node *does not couple* to $\mathbf{R}_4$. Or in still other words, an individual particle (a graph node, which is always one abstracted 'end' of N conjoint unit objects or graph antinodes) does not 'feel' local-real interactions, it sees (directly) only the nonlocal *radial* phase which is its coupling to the inertial mass field - so that this field would be 'flat' everywhere in the absence of local forces, which provide (constitute) the transverse component of $\mathbf{R}_4$. Clearly this is the same as saying that the nonlocal radial phase of the mass field - the proper connection lying on the null signal line between two particles - is itself null (i.e., the *bare* masses m_1 and m_2 in $F = m_1 \cdot m_2 / G$ are zero), transforming as a regauged *zero-point* of mass action in the ensemble, just as states representing transverse local components of the total PM field are felt and mediated only by the *ensemble* of all particles in $\mathbf{R}_4$. So it is $\mathbf{R}_4$ (the special-relativistic field of electric and magnetic potentials) which, considered as decohering a nonlocal supersymmetric anyonic phase to locally coupled bosonic and fermionic particle phases, simultaneously renders the mass field non-flat and produces gravitational potentials.

conditions of this process demand that unit scale acquires new, specifiable, relativistic local variables in superposition, thus transforming instead to real $\mathbf{R}_4$. Treating this as a transformation of $\mathbf{C}_N$ (rather than as an annihilation of it) the effect of accommodating this transformation is the same as if unit scale expands (acquires positive real values) on the dimension normal to the $\mathbf{F_2}$ hyperplane, a 'thickening' of $\mathbf{F_2}$ that hides the primitive identity of all scales as a broken symmetry (a nonlocal phase) *inside* its new $\mathbf{R}_4$ (local) transform.[71] The Lorentzian local symmetry of $\mathbf{R}_4$ becomes the carrier of the hidden $\mathbf{C}_N$ symmetry, and the inflated hyperplane expresses 'inside' it as a scale-free correlation 'length' such that $\mathbf{R}_4$ then behaves with fractal-2 'reduced dimensionality' in respect of a nonlocal spin entanglement of its *N* nodes of measurement.

6.) For very large *N* this expansion will tend to approximate the positive pressure of a continuous true vacuum, or one might say that the local-real domains ('objects') that are coemergent with mass exert *negative pressure* and therefore 'attract' the $\mathbf{R}_4$ matrix in which they are embedded. From this point of view local structures can be characterised as *false vacuum* states which have a higher zero-point energy-density (see para.7) than the true vacuum, which latter therefore attempts to displace them. So local gravitation is to be the reciprocal of this global *inflation*. But because this process takes the fractal geometry out of the complex $\mathbf{F_2}$ hyperplane, the metrical projection of the hyperplane is on this account *not* (at *any* emergent scale) a regular lattice; and so in this distinctive sense it is the very presence of gravitation in PM which itself demands that 'non-flat' spacetime cannot be validly approximated as isotropic and homogeneous on large emergent scales.[72] Contrarily, only the notional *absence* of gravity would permit an approximation to global isotropy and homogeneity in the form of a regular and (absolutely) rigid lattice of points; but these would not then be points of *measurement*, which is why for real observers this self-inconsistent state is

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71 We will suggest later that the attempt to conserve equipartite (absolute) unit scale in this process fails at a *numerical* threshold associated with simple polyhedra of PM dimensions, beyond which the avalanche of period-doubling necessarily proceeds by the equipartition of *action.* This *action constant* occurs because PM exclusion expresses as a Lorentz-invariant lower bound on SR interval, whose 2-dimensional transformational invariant is the action (thus we will develop the idea that SR transformations embody rotations and multiplications of a unit vector associated with a unit action and recovers the complex vector space of QM in an extremal limit). The resultant embedding of an $\mathbf{F_2}$ symmetry phase inside emergent $\mathbf{R}_4$, associated with 6-complex-dimensional hyperstructures, may be connected to a characteristic dimensionality of superstring/M-theory.

72 This suggests an illuminating view of gravitational entropy. If $\mathbf{F_2}$ is a surface encoding the information contained in its projective equivalent ($\mathbf{R}_4$) then all possible configurations of $\mathbf{R}_4$ are equivalent implications of the same cosmic informational limit, making the $\mathbf{F_2}$ lattice entropically analogous to a black hole horizon, where the *Beckenstein bound* on the information encoded on the surface area of a black hole is proportional to total area divided by the Planck area. This result is extended to cosmological 'black hole' entropy in quantum gravity theories, but it is problematic that the possible number of Planck volumes inside the horizon exceeds the number of Planck areas on the horizon for the same quanity of information. This is resolved in PM because, away from the limit, the "Planck scale" is *not a unique length*, but goes to all possible scales on the radial mode of the inflationary projection from $\mathbf{F_2}$ to $\mathbf{R}_4$. This connects distantly with the idea that a Planck-length spacetime grain size is "blurred" or smeared to about 10^{-16}m in a holographic quantum gravity theory, but whereas transverse holographic noise is predicted by such theories due to Planck blurring of some 19 orders of magnitude (c.f., Craig Hogan, Phys. Rev. D 77, 104031 [2008]), PM predicts smearing of the (radial) grain by *all* factors up to 60 orders of magnitude (or the cosmic scale ratio) which is the PM inflaton mode.

unstable, and why (paradoxically in terms of the entropy of spacetime) a state of null gravitation is a far-from-equilibrium condition. Equipartite (absolute) scale must decay into relativistic scale in order to *generate* measurement of real interval (time and potential energy). To describe the 'presence of gravity' is to describe the transformation of a scale-free hyperspace lattice on which unit interval is an indefinite **basis state** for an action, to an anisotropic, inhomogeneous and irregular real network on which intervallic action is (relativistically) definite. Thus gravitation, as the *generator* of real times and scales, is to be also the *agent* of the quantum condition itself, which is to be emergent in the breaking of the anyon supersymmetry.[73]

7.) In PM the meaning of rational length scale itself is connected with the interpretation of mass as an inherently distributed (nonlocal) property belonging to an *ensemble* of unit objects. This mass is formally identifiable with the *gravitational potential energy* of the ensemble. Clearly a gravitational potential has no rational meaning in terms only of an abstracted pair of points, and neither does a *measurement*. In discussions of relativistic measurement, an idealisation is sometimes assumed as though real position/momentum measurement of one point particle was possible from within the frame of another isolated point particle, but the abstraction is unrealistic.[74] In the real world, any length specification (measurement operation) in $\mathbf{R}_4$ involves the included angle(s) subtended by displaced spacetime origins in an *ensemble* or system of nodes, and this is just the condition mandated foundationally by the PM exclusion principle which says that a degeneracy of measurement points (nodes) is forbidden by the nature of the theory's primitive unit object. A non-degenerate ensemble of N nodes obviously cannot be the origin of a singular radial (lightlike) phase of the network, but can only be an approximate superposition of N slightly displaced origins constituting a reference body or measurement system (or, in different language, measurement occurs where the wave phases are in general incoherent). So to say that c is confined on the longitudinal photon phase of a *many*-origin state space for such a measurement system is also to say that the *zero-point basis state of spacetime interval* (or unit scale) is properly *renormed* from node to node of the system, which can be interpreted by *im*proper measurement as a renormalisation of c, and thence as proportional to a curvature of the SR light gauge across a gravitational potential gradient.

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73 The gravitational *neg*entropy of $\mathbf{R}_4$, acquired in the inflaton symmetry-breaking, is thus its quantum signature, and the Planck quantum condition for cavity radiation derives ultimately from a *blackbody spectrum of metrical scales* (predicted to be a universal fractal property of the mass field) throughout the cosmic cavity, skewed from a Gaussian curve in the breaking of the primitive symmetry of anyonic *unit* quanta to a dual-viewed *binit* symmetry (see *Note 68*) wherein the emergence of transverse field components represents the decoupling of matter and radiation fields.

74 This recalls objections raised by Exner, Schrödinger and others early in the development of quantum theory that classical determinism - assuming the possibility of precise knowledge of particle position and velocity 'at an instant' - had always been an illusion encouraged by abstraction, because multiple successive operations are intrinsic to any act of velocity measurement.

8.) Obviously in ordinary Euclidean 3-geometry the transverse projection of a 1-dimensional rod of some irreducible finite length only becomes zero for any observer either by going to infinity (impossible in less than infinite real time) or by a foreshortening rotation to the radial orientation of a ray in the line of sight. This rotational limit is trivial in a homogeous and isotropic Euclidean space. But an analogous rotational limit on scale transformation becomes structural in the Minkowski 4-space of SR with its +,+,+,- spacetime signature, where velocity transformations are naturally the sums of hyperbolic tangents. The "speed of light" represents a limit of just such a rotational transformation. Multiple non-parallel Lorentz transformations or *boosts* produce not only relative velocity changes but relative object rotations (Wigner rotations). The real Lorentz contraction of an ideal rod moving transversely to the observer's line of sight and subtending a small angle in the direction of motion (i.e., a rod that is physically very short and/or remote, such that light rays reaching an observer from both ends of it are approximately parallel) is also generally a perspective rotation (Penrose-Terrell rotation), and behaves in the same way, the rod becoming foreshortened towards a radial orientation in the limit of *c*. This limit is functionally equivalent in $\mathbf{R}_4$ to the unreachable limit of infinity in Euclidean space. This function of *c* in SR (i.e., in the notional absence of gravity or on "infinitesimal domains" of the metric tensor space in GR) can be shown to be expressible without reference to classical or quantum optics as simply a constant of the transformations to be determined by experiment.[75]

9.) In PM, in the same spirit, the existence of a limit with the character of *c* comes ultimately from our exclusion principle and represents the obvious fact that a transverse projection of unit scale at A *never* goes to real zero, by definition, except by *ceasing to be transverse* at A. It may not be transformed away by an absolute shrinkage (forbidden by PM exclusion) but only by a rotation to a radial element of the "sky" of node A, a rotation which must inherently involve transverse *non*-zero specifications within the displaced frames of other nodes B, C . . . *N* constituting the measurement system. That is, the nature of the process of real measurement (self-transformation) on the network itself dictates, by definition, that there shall be a limit of measurable relative velocity for a "particle" (by which one means here "a node") equivalent to a limit of angular rotation, involving a transverse displacement of position states constituting a measurement frame (of other nodes), where displacement is in turn proportional to a gravitational potential.

10.) Considering in the abstract the "point of view" of one node, A, we can see that any transverse projection of unit scale thus transformed away to a radial element of the "sky" of A must then lie on a radius from A containing at least two further collinear nodes, B and C, and may be transformed

75 Mermin, N.D, *Relativity without light*, Am. J. Phys. 52 (1984) 119-124; Feigenbaum, M.J., *Relativity - Galileo's Child* http://arxiv.org/abs/0806.1234v1

back to an element with a transverse component at A by displacements within the ensemble. However that set of radial network components immediately coordinate at A and connecting A with the set of nodes lying at unit distance from A (i.e., the set of *N* "one-hop" connections) obviously are *not rotationally transformable* to a transverse component for A, and together comprise a *priveleged metric*. Equivalent privelege attaches to the first-order radial elements also of B, C . . . *N*, meaning that the plural complete space is many-centred projective and that the complete translational symmetry space containing the Lorentz transformation group (for SR this is normally the Poincaré symmetry) is for this reason *not an homogeneous and isotropic space.*[76] These sets of radii form special domains inside the complete PM space, analogous to SR "infinitesimal" domains inside GR space, but they exist over all possible rational scales (all possible *im*proper transforms of properly-null unit scale) rather than none. It is these interpermeable domains that may be thought of functionally as a type of "matter particle" of the PM field, whilst the nodes at their coordinate origins cannot validly be abstracted - i.e., the particle *is* substantially its coupling and is only an aspect of the complete field structure, which is in turn made of the couplings of all particles. This radically dual space structure will also become important momentarily for understanding the particle self-energy.

11.) A radial proper alignment is only available from a single node (= particle-like subspace origin) on a line of sight lying through another single node (another subspace-origin), which of course identifies this radial unit object as a photon null signal line in QED where a single real photon emission is associated with a single electron transition. Simultaneous multiple (real) photons on multiple null signal lines, coordinate at a single electron, do not occur, so it is the essence of $\mathbf{R}_4$ that measurements by means of light signals are not available to a single electron at an instantaneous spacetime location - there are only serial signalling operations by a single electron at successive (space)time positions (in a free electron gas), or parallel signalling operations by multiple electrons at adjacent space(time) positions (in a bound crystal configuration). Both of these states are displacements identifiable with a gravitational potential energy (= total gravitational mass-energy) of the measurement system. Real scale measurements in $\mathbf{R}_4$, and gravitational potentials, thus in effect forbid one another to go to zero, and any unit object which is the pure null component of the radial phase of the network in the frame of any one node must have a non-null transverse component in the frames of every other. All of which illuminates from a novel direction

a) the stipulation that only the theory's massless gauge boson can "travel at *c*" and

b) the problem of the need for a length scale (momentum) cut-off in QED

76 Some approaches to quantum gravity such as "doubly-special relativity" (DSR) propose a deformed Poincaré algebra as a high-energy limit of the low-energy semiclassical Poincaré space.

12.) All relativistic quantum field theories have required a length scale (momentum) cut-off to be introduced, to allow renormalisation group methods to be applied to tame infinite divergences arising in calculations of the self-energy of particles in quantum field theory. This is an *ad hoc* artifice in the homogeneous and isotropic background spacetime underlying these theories. The problem arises because of two factors: The existence in the theory of free singular particles and an homogeneous and isotropic background space (which are reciprocal implications and can be considered as one factor); and the existence of a non-zero bare mass associated with such "matter" particles in QED. Whilst the photon represents only a configurational (binding) energy in QED (i.e., it has no rest mass, only momentum) electrons are said to have a non-zero mass coupling, a theoretical intrinsic or bare energy belonging to them independently of their physically-measurable "dressed" energy, which latter always has a contribution depending on virtual loop terms and interaction energy. Our interpretation *via* PM extends to the idea that neither does an electron have intrinsic or bare mass-energy, because "an electron" is intrinsically not bare (or is essentially *ex*trinsic).

13.) If we *abstract* the extremal case of the "sky" of a single node as mandated by PM exclusion this could be thought of as a projection of nodes lying on a sphere at *unit radius.* This structure is an abstracted "special domain" or subspace of the PM configuration space in the sense of para.10. We treat the realised surface of the unit sphere as representing a projection of the unstable critical-point $\mathbf{F}_2$ hypersurface, with unit radius embodying PM unit scale.[77] To deliberately evoke the machinery of critical-point models, we call this a unit resolution scale or 'lattice' spacing (we can unwrap the projection back to $\mathbf{F}_2$ to imagine this), which is transformed in $\mathbf{R}_4$ so that the unit sphere becomes the nonlocal basis of an N-dimensional hypersphere whose N different radii each have N different local-real specifications in superposition. This hyperfigure now has no typical or average resolution scale, which may be interpreted to mean that the underlying quantum structure of the $\mathbf{R}_4$ metric has *no smooth homogeneity scale* and therefore may not be validly modelled by GR (which of course does not recognise any structure underlying its smooth metric and is not a quantum theory). Instead we say that the gravitational coupling which may be considered to be confined (like the charge

77 This special case picturesquely shows PM exclusion as functionally equivalent to a Cosmological Constant, introduced by Einstein to prevent mass singularity. No such singularity has ever existed in the Einstein model, but the *status quo* is unstable and "wants" to be singular in the future, i.e., it should collapse. Thus the negative-pressure solution of the CC, due to an intrinsic energy of spacetime which turns out to be formally equivalent to the quantum vacuum energy. The necessity for this CC was apparently removed by the expanding Friedman model in 1922 which had no singularity in its future. But it transpired that it did have an inevitable singularity in its past (as was finally proved by Hawking-Penrose theorem in the '60s) which from a certain point of view is just as bad. The CC has thus reappeared and finds a new home in quantum theoretical treatments of cosmic inflation. But the theoretical CC value according to such models is staggeringly adrift from observation by some 120 orders of magnitude. From the point of view of PM this takes us directly back to a fundamental rethinking of the nature of quantum vacuum states.

coupling or photon) on the radial connection between any "pair of electrons" (nodes) is also primitively a zero-point potential, importing into $\mathbf{R}_4$ ghosts of the gravity-free flat state space[78] of the $\mathbf{F_2}$ lattice, and these zero-points are *differentially* regauged by the breaking of the unstable $\mathbf{F_2}$ hypersymmetry (reversible unit lattice) to the emergent SR space of Lorentz transformations (or, to express this in a different way, the electromagnetic field is the gauge of an emergent field of gravitational potentials).

14) Each of the *N* unit radii of each of *N* unit spheres is transformed to a graph connection in local $\mathbf{R}_4$ with N^2 different metrical values in superposition. The unit radius donated from $\mathbf{F_2}$ is preserved in $\mathbf{R}_4$ as a unit radial *action,* a common extremal minimum action underlying *N* transverse relativistic action specifications. The unit sphere carried into $\mathbf{R}_4$ by these action *radii* is the cosmic holographic surface ($\mathbf{F_2}$), of which there are therefore *N* instantiations interpenetrating inside $\mathbf{R}_4$, and the emergent definite extremal value of the unit action *radius* becomes the common invariant of SR and QM. From here we can understand anew the issues of renormalisation, the particle self-energy and the underlying separator of real from virtual terms in the path integral.

15) The unit zero-point identity (inherited from $\mathbf{F_2}$) is a system critical point. To recover the system critical point in a quantum field theory of point particles involves a theoretical proceedure to generate a renormalisation flow, and thus drive the system mathematically towards a critical point, normally by varying a global energy scale. The result then becomes calculable by allowing the (infinite) particle self-energy to effectively drop out, leaving a bare particle mass. But it is not clear in QM that this proceedure, which involves importing a length scale cut-off in a background space (perhaps near the Planck length), has a natural physical justification. In PM, however, the inflationary projection of $\mathbf{R}_4$ from $\mathbf{F_2}$ *embodies* or reifies this renormalisation flow. The system critical-point *follows* the flow to all of *N* energy scales, so that on the gravitational phase (mass phase) the zero-point and the correlation length are identically preserved underneath all improper transformations of unit radius.

16) The interpenetrating matrix of deformed unit spheres - collectively the gravitational phase of the PM space structure - identifies a critical-point behaviour. The analogy of a foam of (nonlocal) bubbles is not a trivial one, since this type of behaviour is physically characteristic of phase

78 This may seem paradoxical since these ghosts are to be the very origin of gravitation owing to the inflationary function of the "hidden" absolute unit scale $\mathbf{R}_4$. Of course we need to distinguish the *force* from the *field.* Each of *N* nonlocal ghosts contributes unit force radially at node A and is blind to its associated direction or magnitude in the emergent $\mathbf{R}_4$ vector space, so the force is intrinsically a scalar and its resultant linear and flat; but gravitational *field* is a non-flat geometry arising in the *ensemble* of nodes A, B, C. . . *N* where potential gradients are due to the differential spherical *distributions* of unit force emergent *via* the $\mathbf{R}_4$ gauge.

changes. In terms of the standard cosmological model the phase change would be the force-decoupling at the GUT era that releases tremendous energy and triggers a sudden inflation about 10^{-30} sec after a 'big bang', rolling over almost instantly into the sedate Hubble flow measured today by the cosmological redshift. From the point of view of PM the inflation is a property of *all* cosmological epochs, and its signature on spacetime is the scale-invariant zero-point action that is preserved on the radial gravitational (mass) phase. Thus the special radial domains identified in para.13 *et seq* can be thought of as inflated bubbles of a Planck foam, where the inflation has rolled over in each case at N different real scales simultaneously, or each bubble acquires N^2 different improper radii in superposition.[79] And because the PM space structure is (so to speak) surfing the edge of symmetry-breaking from $\mathbf{F}_2$ to $\mathbf{R}_4$, we have a system whose configurational energy is a physical embodiment of the renormalisation procedure, and where in effect, on the proper radial mass phase of PM space that preserves (from $\mathbf{F}_2$) correlation length equal to lattice spacing, *all* scales are cut-off scales.

17) In this structure the particle self-energy is an aspect of the system energy. In QM the particle self-energy is contributed to by pathological off-shell self-couplings. These are couplings of a point particle with itself *via* closed virtual loops that proliferate catastrophically down to ever smaller *real* spacetime scales. But in PM the integration over all possible virtual perturbative terms occurs on the real-null $\mathbf{F}_2$ phase of PM state space. On this phase all fluctuating pathways on the network loop back nonlocally to their origin in unit time, and thus contribute to the resultant radial constraint. This is the mass phase, so that in this sense it is "including gravity" that cures the infinity sickness of QM. One could also say that catastrophe is averted because the *virtual loops* terminating on the radial phase of the network exist only in a *virtual space*, whose only timescale is everywhere unit time and where the calculation is sensitive only to the *number* of terms each of *unit value*. On-shell contributions in this scheme live on the transformed unit spheres which we have identified (para. 10) as special scale-free domains inside the complete space and which are functionally the "matter particles" of the PM field. Off-shell contributions in this scheme represent loops that live on neighbouring unit spheres,[80] i.e. domains whose radial phases are nonlocally (virtually or *properly*) co-original but are locally (really or *improperly*) displaced from one another.

79 This inflationary signature is not an ancient echo but is spread over all epochs, and this is because in PM the 'holographic' horizon surface is not like a monolithic shell 14 billion LY away from everywhere (i.e., not behind the CMB surface as in holographic theories based on the standard cosmological model) but rather is itself plural, deformed and distributed over all scales and epochs. In another analogy we could say that PM space is "shot through" with its boundary condition, or in Bohmian terms that its boundary is *implicate* rather than spatially peripheral. Another way of looking at it, from the perspective that c is renormed at every node, is that every node sits on the light horizon of every other.

80 In the PM complete-graph construction of course the origins of *all* other unit spheres are immediate (one hop) neighbours.

18) Our point of measurement is always a certain node (actually, it is always a relation of some subset of nodes). If we identify this point (or points) with the conception of a "particle" in conventional field theories, then it appears that *all* its self-energy contributions in our scheme are in a sense off-shell, because the mass shell is not local-real and does not exist at the location of the "particle". But it is a *locus* of the particle. That is, we redefine the mass "shell" *not* as the surface of a sphere of arbitrary small real radius (conventionally the "electron radius" for example) but rather as the surface of a figure of unit radius in the state space, which of course acquires all possible real scale values in $\mathbf{R}_4$. Thus the shell surface is defined operationally rather than geometrically. As described in para.8 the basic anisotropy inherited from $\mathbf{F}_2$ means that the complex symmetry of unit scale is only broken for *improper* transverse measurements. So improper-transverse lines (constituting an indefinite number of ≥2-hop off-shell self-energy terms) and proper-radial null lines (constituting *N* one-hop radial terms defining a surface at unit radius which sums the on-shell energy) are qualitatively different and non-interrotateable axes of the first-order spacetime construction. An interrotation is a transformation available *properly* only to ≥2-hop off-shell terms since all the on-shell contributors to the mass are radically radial. This is the separator of real from virtual terms.

19.) So we propose that the acquisition of mass *via* the locally scale-dependent transformation of an underlying scale-invariant critical-point structure in PM can be interpreted as a physical embodiment of renormalisation group proceedure in a flow of scales driven (in a certain sense) by an *inflation* which is the *N*-valued self-interactive transform of primitive unit scale into $\mathbf{R}_4$. This transformation is a breaking metasymmetry in which the complex orthogonal axes of the rigid $\mathbf{F}_2$ lattice are collapsed onto one another, being preserved underneath $\mathbf{R}_4$ as self-orthogonal photon-antiphoton null vectors that represent the shadow of PM unit scale. Unit scale is thus to be a **basis state** for an emergent *unit interval* which will be able to exhibit many real values in relativistic *superposition*, each a spacetime interval associated with a different relativistic specification. This fundamental mode remains real-scale invariant and vitally can be *arbitrarily larger or smaller than any given real scale* in emergent $\mathbf{R}_4$, where it represents the product of unit time and unit potential energy in the PM network. This is why the quantisation of PM gravitation is not an energy-scale-specific limit condition. Unlike the extremal “Planck scale” gravity regime in an effective-field projection, PM gravitational quantisation occurs overtly in the physics of all scales and at all epochs. One pregnant way of looking at this is to say that a quantum of gravitational mass and a quantum of gravitational action are two views of the same scale-free thing.

20.) The superposition of relativistic states in para.19 represents a tiling of triangles that expands until the connectivity becomes that of a complete graph with N lines joining all $N^{1/2}$ vertices, which is what takes the tiling construction out of the $\mathbf{F}_2$ hyperplane to a geodesic N-dimensional volume of complex planes. And this 'curvature' describes the process of how we expect unit scale to acquire definition, in proportion to the narrowing constraint of having to satisfy increasing numbers of angles self-consistently, where all angles are also co-dependent functions of the same constraint. Hence the proliferating number of degrees of positional freedom for acts of measurement (spacetime) is emergent as the reciprocal of a diminishing trigonometrical freedom. The essence of $\mathbf{R}_4$ is the greatest possible redundancy. Indeterminate clock rates (primitively, the indiscernibly identical reiteration of unit time) converge to mutually consistent (discernibly non-identical) actual clock rates. The actual emergent length of any unit scale for any observer can be represented (in the limit of some remnant indeterminacy inversely proportional to the number of observers/origins) as some multiple of this imaginary unit vector which is to be found as a function of an emergent real angle.

21.) Obviously it cannot matter where we start. It is evident that a self-consistent network (i.e., a complete graph) of N lines requires (or has to justify) the use of $N[(N\text{-}1)/2]$ sets of operations at *every* origin in order to supply one real value for each unit interval. As it happens this is very natural to the spirit of path-integral QM, and we propose that the relativistic operation of superposing, on each interval, N real spacetime transforms of a conserved extremal action traces the quantum **commutation relations** of the cosmos. In *Section 2.5* we discussed the case of an abstracted triad OAB and asserted that the physical meaning of the cyclical commutation relations is that each side of OAB operates both in a kinematical and a dynamical role to supply both force and displacement amplitudes. Each pair of force and displacement vectors, in turn, determines a rate of work proportional to the cosine of their argument. The rate of work, or the power, has the same dimensions as *action*. Consequently the sequence of scalar products of successive pairs of vectors, each in these complementary roles, corresponds to the *spacetime ordering* of the momentum and position operators $pq - qp = \hbar/i$. This order appears in the Feynman operator algebra as

$$m\left(\frac{x_{k+1} - x_k}{\varepsilon}\right)x_k - m\left(\frac{x_k - x_{k-1}}{\varepsilon}\right)x_k \underset{s}{\leftrightarrow} \frac{\hbar}{i} \qquad (30)$$

where ε is a small increment of time, like $t_{i+1} - t_i$. So the increment of position $(x_{k+1} - x_k)/\varepsilon$ is a particle velocity, the term $m(x_{k+1} - x_k)/\varepsilon$ therefore corresponds to the classical momentum $p = mv$, and the difference between the two differently-ordered noncommutative products is equivalent to

$\hbar/i$ under some given action, S.

22.) The conjugacy of position and momentum arises for every state in every triad in $\mathbf{R}_4$ because they are both cyclically interdependent functions of emergent real values all over the exhaustively connected space $\mathbf{C}_N$, and in this sense it is the very indefiniteness of unit scale that is the origin of periodicity: The rotating permutation of solutions that is necessary to express the coherent superpositions of $\mathbf{C}_N$ as *decoherent sequences* of real states is the primitive triadic reiteration of 'unit time' that gives rise to clock frequencies in $\mathbf{R}_4$. And we propose that the spacetime ordering of quantum operators (equivalent to the ordering of matrix operator terms in the older quantum theory, of course) is the same cyclical permutation of resultants that we have identified as giving rise to non-zero mass in PM triads. The condition $pq - qp \neq 0$ corresponds to the condition $\boldsymbol{m}_i - \boldsymbol{m}_g \neq 0$ in actual plural systems. As first suggested in *Section 2.2*, any wholly indeterminate unit scale would satisfy $\boldsymbol{m}_i - \boldsymbol{m}_g = 0$, the cancellation of two antiparallel vector operations taken in arbitrary order, or equivalently the simultaneous superposition thereof in *no* order. This state gives the zero-point of (pseudo)*scalar* mass on the anyon hyperplane of $\mathbf{F}_2$. The condition $\boldsymbol{m}_i - \boldsymbol{m}_g \neq 0$ arises only for the *plural* broken symmetry of emergent measurement systems, where the cyclical triadic ordering of the operators brings in the directed periodicity that we call time as determinant of emergent real scale.

23.) The fundamental angular unit of the network echoes the hidden lattice symmetry of the hyperplane and is given by one iteration of a zero-point action $\hbar/i$, where i is seen as a rotation operator for a phasor. In this function i operates, as we pointed out in *Section 2.5* (para.19-24, Fig.23, 24), as a renormalising factor for c, so that moving from vertex to vertex through multiple broken phases of the lightlike line takes us from one 'view' or complex transform of the hyperplane to another with c always normalisable to 1. In this way the radial phase of PM space echoes the orthonormal basis of the infinite-dimensional Hilbert vector space of quantum theory, where every state vector evolves linearly and causally remote from all others until the point of reduction. The lattice representation of each transform can be abstractly represented on paper by the Argand plane, where division by i rotates the zero-point unit vector clockwise through 90° so that this plane returns to itself through 4 rotations. The PM triad that physically embodies this complex operation for each real measurement returns to itself through 3, and the action goes to zero only three times. The 4th turn that completes one rotation of the complex phasor represents a move out of the triadic planar instantiation of $\mathbf{F}_2$ into relativistic $\mathbf{R}_4$ and thus represents emergent time.

24.) The abstracted triad embodying the time-independent case can be seen as containing the *potentiality* for time-dependent real measurement in its self-cancelling antiparallel vectorial structure. A progressive wave on a continuous loop, as on an infinite open string, produces no change of momentum, preserving the linear character of unit scale (rotational symmetry of congruent transformation) indefinitely. But any self-interacting (knotted) *network* must contain (at least) closed subnetworks[81] enforcing the existence of wave antinodes separated at wave nodes corresponding to changes of momentum. In the abstract, the simplest such subnet is a triad of antinodes, whose every 3rd rotation returns unit scale to itself with an *inverted phase and direction but withour specified real momentum.* It is a stationary state with no definite energy (i.e., it is not an eigenstate and remains a coherent superposition of all possible wave modes).

25.) If we regard this irreducibly primitive triadic subnet as the flat template of (classical) 3-space on which $c = 1$, then a *tetrahedral* subnet represents the emergence of relativistic specification by a fourth node. The co-emergence of the conditions $pq - qp \neq 0$ and $\boldsymbol{m}_i - \boldsymbol{m}_g \neq 0$ means that the rotation of the time-independent stationary state has become the commutation cycle of a quantum system containing both *potential energy* (real scale, cognate with mass) and *time* (because rotation now produces not merely a real-null phase inversion but a rudimentary clock rate). Thus decohered the stationary condition acquires a definite energy eigenstate, expressed as an increment of real action, where a new condition $c \neq 1$ can be represented as a curvature of a non-flat (relativistic) 4-space now 'including gravity'. Thus the quantity c is proportional to unit angular momentum in the abstracted flat triad, which is an instantiation of $\mathbf{F}_2$, and becomes proportional to a real angular momentum in $\mathbf{R}_4$.

26.) This is a *difference* of real action of course, or Δs, which goes to a zero-point where $pq - qp = \Delta s = 0$ as the average or *expectation* value of the anyon mode on $\mathbf{F}_2$. This special case recovers the hidden $\mathbf{F}_2$ hypersymmetry in the form of transverse pseudo-boson correlations, such that *all* pairs of fermion states therefore correlate bosonically some *proportion of the time* for all appropriate observers. The proportion will tend to 1/3 as the notional probability of measuring such a correlation in the lowest positive real energy condition for an isolated ideal triad. We can describe this fraction as a large coupling constant; but an isolated coherent triad knows only unit time and contains no real clock, so 1/3 cannot be a proportion of real time for any real system and does not represent a real photon probability or a real action. But we can speculate that the experimental charge coupling constant - about 1/137 - ought to be found as a perturbative correction to a

81 And the PM complete graph is of course a closed network composed of such (approximately) closable sub-networks. Note that these closed nets must exist *inside* the domains which we have identified as the functional mass shells of PM "particles", which reilluminates the issue of "quantum hidden variables".

theoretical unperturbed virtual photon exchange probability of 1/3.[82] One implication is that asymptotically approaching the minimum energy state of a system of charges by shielding it from interaction (i.e. from thermal and magnetic disruption) would recover an approximation to the unperturbed large coupling constant in the form of enhanced pseudo-bosonic charge couplings. (This may be relevant to the problem of superconductivity.)

27.) The lowest-energy electrodynamical state of a triad with the lowest probability of photon emission approaches the ideal condition where all states are anyonically-correlated. The complex zero operator then corresponds to a minimum energy value of the permutated operator ordering and oscillates around positive and negative states, so that Newton's classical ***F*** - ***ma*** = 0 actually corresponds to the *average* value of a *non*-zero operator, rewritten in terms of Eq 30 as

$$0 \underset{S}{\leftrightarrow} -\frac{m}{\varepsilon}\left(\frac{x_{k+1}-x_k}{\varepsilon}-\frac{x_k-x_{k-1}}{\varepsilon}\right)-V(x) \tag{31}$$

where the term in brackets over ε represents the acceleration ***a*** and the derivative of the potential -*V* (x) corresponds to the force term, ***F***. There are both low- and high-energy boundary conditions. The *highest*-energy electrodynamical state with the *highest* probability of photon emission *also* approaches the ideal condition where all states are anyonically-correlated, which is to say that the true zero operator and the infinity operator are equivalently inadmissible as actual states.

28.) In this sense all triadic real relations express the average expectation values of false-vacuum states due to a finite, non-zero operator, but occur in the embrace of a universal correlation associated with both imaginary extrema. So because structures of actual relations, i.e. 'objects'[83], *are* false vacua which, we say, incorporate the emergent operator for inertial/gravitational mass, it

82 The experimental virtual photon emission probability associated with electron charge is found to be 1/137.03597 (the fine structure constant). We conjecture that this value is a limit set on the $\mathbf{R}_4$ network by the minimal tetrahedral hypersymmetry group embedded within it (see *Note 3*) whose 6D structure is a numerical limit of equipartition of absolute unit scale (analogous to a classical limit, as it were) inside relativistic $\mathbf{R}_4$. Each possible triangulation of unit interval produces *ideally* a photon probability or "coupling constant" of exactly 1/3, but considered perturbatively 1/3 can be approximated as the convergence of a power expansion series within this group. Rotating unit interval around the tetrahedron returns it to itself after five iterations, and the 5^{th} power of 1/3 is 0.0013716, which is approximately the reciprocal of alpha x 10^{-5}. We suggest that an isolated tetrahedron emulating $\mathbf{F}_2$ would embody an ideal non-perturbative alpha corrected by this one part in 10^5, but such ideal coherence cannot be realised in $\mathbf{R}_4$ (the essence of observation/interaction being of course decoherence). The lower bound of quantum-cosmic departure from classical scalar smoothness in $\mathbf{R}_4$ ought therefore to be an intrinsic vectorial roughness of 10^{-5}. Interestingly 10^{-5} is the observed order of inhomogeneity in the CMB.

83 These mixed-state actual structures might better be called pseudo-objects, to be distinguished from perfect objects. Perfect objects would be pure proper states, just as perfect relations would be pure improper states. Such pure states are not actual; they are *singularities* of the PM geometry. An isolated PM node would be such an unintelligible pure state. But the actuality of pseudo-objects consists in their being *mixed* proper/improper (i.e. relativistic) states just as they are also mixed real/imaginary (i.e. complex) states. We use the word "objects" to mean such mixed, complex, relativistic pluralities.

follows that these structures *are* functionally 'Higgs fields' where an inflationary supersymmetry is broken locally and coemergently with mass and gravitation. But the notion of a Higgs field having a value at a point in a continuum is replaced by the value of a mass operator m_i - m_g, which is non-zero in actual *plural* systems made of triplets of PM doublets. So we can say that the Higgs-type PM mass operator remains a zero operator inside any *single* doublet, and recovers zero as an 'average' value in the sum over *all* doublets. Between these unrealisable extrema, the false vacuum zero-point masses of objects are equivalent to their local-real relativistic momenta.

29.) Actual objects (structures of points of measurement on arbitrary real scales) are acts of complex interrelation, acts in which *ratios* of quantities appear and are registered. The condition $\Delta s \propto \Delta m \neq 0$ occurs inside systems composed *both* of massless null radial *and* of massy non-null transverse components. To repeat: The included angle at the vertex of *two* components conjoint with a *third* component is integral to the physics of 'observation'; the inclusion of a determinate third angle is equivalent to 'breaking' the PM geometrodynamical supersymmetry of boson and fermion (what we have now called the critical-point anyon supersymmetry) and the process is an irreducibly plural and *mutual* activity involving an exchange of roles from vertex to vertex which rotates the 'mass' around the triad as the scalar product of three different pairs of vectors. This rotating construction underlies the quantum commutation relations in PM and depends on the fact that each of these vectors (complex vectors in a hyperplane of self-orthogonal real and imaginary coordinates) operates both in a kinematical and in a dynamical role to supply both force (momentum) and displacement (position) amplitudes. Thus the triad is the minimal symmetry group for the emergence of the dynamical quantity called 'mass', which is null in the superposition of antiparallel force amplitudes contra-rotating with equivalent opposite sign ($\pm t$) in the anyon supersymmetry, but which, away from this critical equilibrium in actual conditions of interaction, emerges as non-zero in the separating out of electron and photon aspects of the underlying PM unit vector.

30.) In QM arbitrary complex vector rotations in the state space are permitted in preparing the quantum state. The probability of realising the final state vector is dependent only on a *ratio* of phases in the complex plane which themselves have no measurable physical significance. The real state is the product of the special symmetrical pair

$$\mathbf{r}(\theta) = \boldsymbol{e}^{i\theta} = \cos\theta + i\sin\theta$$
$$\mathbf{r}^*(\theta) = \boldsymbol{e}^{-i\theta} = \cos\theta - i\sin\theta \qquad (32)$$

of a state vector **r** and its complex conjugate **r*** mirrored on the *-i* side of the real axis. The phase itself has no physical significance in QM, just as the anisotropic complex plane has no immediately obvious significance in an isotropic and homogeneous real Lorentzian space of $\mathbf{R}_4$ which is designed to preserve equivalence of all possible coordinate axes through arbitrary rotations for inertial transformations. But famously neither QM nor this special-relativistic isotropic real space "include gravity".

31.) That is, Minkowski $\mathbf{R}_4$ is known to be incomplete because, in some sense, reference frames that are in gravitational field *do* introduce privileged directions. The point of view of GR is that the privileged directions represent a distortion of isotropic and homogenous $\mathbf{R}_4$ which in turn reflects its coupling to an inhomogeneous distribution of mass-energy. This is perfectly intelligible only as a description of a co-dependency, not as an explanation. An $\mathbf{R}_4$ conceived as a free background that might in principle not have been coupled to a mass-energy-momentum tensor and might therefore have been flat is an intrinsically unobservable metaphysical entity. Our 'observation' of $\mathbf{R}_4$ occurs only *via* the gravitational distribution of mass-energy that constitutes $\mathbf{R}_4$, and this physical mapping *is* the physical territory. So the existence of mass-energy is not an answer to the question: 'What is the cause of the fact that the spacetime continuum is not a flat, minimum-energy configuration?' Another way of asking the same question is: 'Why does GR spacetime have a singularity in it?' In GR mass and singularity imply one another.

32.) The point of view of PM is that mass-energy and spacetime structure *are* the same thing. There is no free background and no singularity. Not only does the space structure generate mass in the absence of actual singularities, it is *the very absence of real singularities* in its graph structure (PM exclusion) that *forces it to generate mass*.[84] And the reason proposed in PM for the incompleteness of special-relativistic $\mathbf{R}_4$ is that complex phase *does* itself have a physical representation; that the phase rotations are fundamental, the spacetime field an effective representation; and that to "include gravity" will be to show how $\mathbf{R}_4$ is built out of transformations between arbitrary complex rotations. The core of the idea that $\mathbf{R}_4$ embodies ratios of complex phase depends on the idea that the primitive PM unit scale acquires the character of a real vectorial interval as a simultaneous superposition of *N* complex states, each of which represents one of *N* rotations in $\mathbf{R}_4$.

33.) PM proposes that in a system of *N* measurement points[85] every triad of points superposes *N*

84 We have said that the graph nodes are singularities of the PM geometry, but they are not *actual* singularities of the network *structure.* PM *space* would only contain singularities if two nodes were permitted to co-locate, but this is forbidden by the exclusion principle in our foundational definitions. See also para..25

85 Measurement of course being a self-interaction of the system registered by the system as a difference between at least two states of itself and having definite rational meaning only for subnets of >3 nodes. Each point of measurement

measurement-constructions like OAB in *Fig 29* each belonging to one of *N* different frames. As set out above, considered in abstract isolation the lines like moduli OA or OB in each such construction are scale-free **basis states** for **intervals**, expressing the radical “exclusion principle” of our foundational definitions in the form of unit scale. Considered as unfilled primitive states they have no well-defined complex phase and no amplitude at all. Dually, considered as ideal filled states they are superpositions of *all possible* complex phases and amplitudes. Considered as states actualised in the self-interaction of the complete PM graph of order *N* they are superpositions of $N^{1/2}$ complex phases and amplitudes.

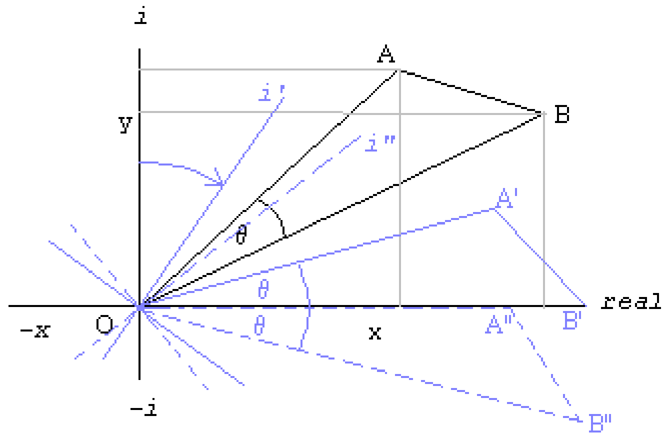

Fig.29. A rotation of coordinates superimposed on an arbitrary complex angle OAB so that OB and OA respectively become pure real in frames *i'* and *i"*. The PM space structure produces R_4 from the F_2 hyperplane containing *N* frames like frame *i*. We can say analytically that on F_2 the vector OA (or OB) time-independently superposes *N* indiscernibly identical phases, whereas in time-dependent R_4 the superposition decoheres to pure real and pure imaginary phases in complex inter-rotation like antiparallel vectors +*t* and -*t*.

34.) A physical picture of this basis state would be unit wavelength on a string of unit length, a closed stationary condition where two equal and opposite wave progressions sum everywhere to a zero-point amplitude (i.e., opposed virtual travelling waves cancel to real zero) which is simultaneously both the fundamental mode *and* the Fourier synthesis of *N* partial modes. Any generally non-periodic or periodic directed function $f(x)$ on the network (with emergent $+t$), having arbitrary length and complexity, is analysable into a series of N^2 non-periodic functions over the limited ranges $-\pi < x < +\pi$ between successive nodes, each range in turn being analysable into a series of *N* simple harmonic terms.[86] The properly-preserved underlying fundamental mode (null, flat,

is represented as the state of a 'particle'.

86 The designation as periodic or non-periodic is ambiguous. The whole network, considered as a single recursive string, must be regarded as a simple harmonic periodic disturbance travelling equivalently in the *x* or -*x* directions with $c = \infty$ and $t = 0$. Smaller, well-defined, bounded domains of the string are non-periodic functions living on series of antinodes with $c = 1$. Strictly speaking each of the N^2 limited-range functions between nodes is non-periodic in one real time direction (i.e., does not repeat to infinity beyond its range) but is internally complex-periodic - i.e., infinitely repeating - in two +/- time directions.

zero-point, real time-free and real scale-free) transforms to an actual magnitude in the ensemble, where real interval emerges as an improper function of relativistic trianglulation.

35.) As does Cramer's Transactional Interpretation,[87] PM takes seriously the fact that the Wigner time reversal operator is complex conjugation, or complex phase reversal. This means that reversing the complex phase of the state vector (reversing the sign of the imaginary part) consistently reverses the signs of time, energy and frequency observables, so we interpret the Born probability law **P = ΨΨ*** to mean that the probability of an outcome is found by taking the product of a component of the state vector with its time-reverse. In TI this operation is said to collapse the state vector at the conclusion of an atemporal "handshake" between two particles, but TI says nothing about the underlying reason for this operation or its relationship to the spacetime structure (although it requires a relativistically invariant wave equation).

36.) The idea is that PM should reify, as an actual operation in its cosmic space of complex vector triangulations, the QM proceedure of producing the wave function **Ψ** with its complex conjugate **Ψ***, taking the projection on the real axis of the state vector diagram, and associating the square root of a probability to the resultant vector amplitude. But to understand the function and cause of the transaction we invert the priorities of the QM causal order, placing the global nonlocal network state in a causal relation to the local transactional outcomes whose frequencies are conventionally called probabilistic. Each self-orthogonal lightlike interval is continually switching complex phase, like a continual exchange of real +*t* photons and imaginary -*t* antiphotons, in response to the conjugation operations ongoing all around the network, driven by its neighbours and ultimately by the self-interaction of the complete graph. A rate of virtual (proper, coherent) phase-reversal transaction underlies measured (improper, decoherent) changes of state proportional to a real coupling strength, not expressing some spontaneous magic called "chance" but rather indexing locally time-dependent states of a pseudo-random (i.e., indefinitely complex) nonlocal and time-independent dynamical feedback process.

37.) The PM space of networked basis states is an $N^{1/2}$-centred super-space of $N^{1/2}$ interpenetrating N-dimensional Hilbert spaces centred on N origins like O.[88] On any radial OA the interval ($t \cdot -t$) is properly null (a lightlike and Minkowski self-orthogonal basis state) expressible as the two phases +*t* and -*t* coherently superposed as exactly-cancelling antiparallel vector magnitudes. For an ideal

87 http://www.npl.washington.edu/ti/TI_toc.html

88 See para.10 These special domains of the state space comprise the set of N "one-hop" radial network components immediately coordinate at every O and connecting O with the set of nodes lying at unit distance from O which, being *not rotationally transformable* to a transverse component for O, together comprise a priveleged metric.

"observer" at O this complex state of OA, properly independent of any projection onto an actual axis in $\mathbf{R}_4$, is a collapsed instantiation of the $\mathbf{F}_2$ hyperplane, a null complex-planar state vector like $\mathbf{\Psi\Psi^*}$, expressing coherently in the symmetry of ($t \cdot -t$). As such it is only a unit-length basis state for a probability amplitude (lying on some ray of the *N*-radiused unit hypersphere original at O). But in the improper-transverse transformation which attempts to construct relativistic $\mathbf{R}_4$ from *N* instantiations of broken $\mathbf{F}_2$ each OA is forced to acquire *N* non-null relativistic magnitudes in superposition, each a complex-planar state vector where the symmetrical conjugacy of $\mathbf{\Psi\Psi^*}$ is *deformed* for a measurement made at B, C, D . . . *N*, producing $pq - qp \neq 0$ (with $\Delta t \neq 0$) on every other real axis. So the real interval is composed of an absolute magnitude preserved as a complex ratio inside a metrical interval, which ratio relates metrical distance to probability amplitude, potential energy to a realised transform of unit time, Coulomb force to charge, and gravitational force to mass-energy.

38.) That is to say, we are turning around the idea that QM amplitudes control the probability of a particle being found at place A in a background space, and saying that QM amplitudes control the probability that there *is* a "place A", i.e., they control the structure of a space which is only manifest in terms of the relationships of observables like A, B, C . . .*N* that collectively triangulate the meaning of "place". Because mass is an emergent ensemble property we can interpret the process in Penrosian terms as mass controlling the objective reduction of the state vector. Equivalently, we can say that the ensemble performs each state vector reduction on a deformed Argand plane where conjugate moduli acquire different real and imaginary amplitudes, and where the unit circle containing all probabilities summing to p = 1.0 is deformed to some type of unit ellipse, enforcing non-null improper projection in which complex phases do not cancel away. The tiling of these deformed planes constitutes effective spacetime curvature.

39.) Mass *is* potential energy - binding energy - and gravitational potential energy is on this account sometimes regarded as a redundant invention, an unobservable, which ought to be replaced always by the supposedly more objective and ‘observable’ quantity mass. But we say, inversely, that this equivalence actually means that mass can only be emergently well defined in the ensemble of interpenetrating and interlocking unit-radius mass shells, echoing (in the limit) the Machian point of view that 'particle' inertial mass *is extrinsic*, a function of one particle's (or node's) relation to all other particles (or nodes) in the cosmos at large.

40.) Arguably some implementation of Mach's principle has to be recovered in classical cosmological solutions of GR, but it has not been clear how this makes contact with the non-classical quantum Higgs mechanism for the origin of mass in particle physics. In terms of the latter, Higgs field and gravitational field are different entities. The particle inertial masses acquired through interactions with Higgs bosons are regarded as a type of charge which then governs the strength of the particle's coupling with the gravitational field. But the awkward partnering of these two mechanisms does not fully explain the equivalence of inertial and gravitational masses. A natural relation between the Higgs lattice and the gravitational field (or between Higgs boson and graviton) seems to be missing, and this can be interpreted as a symptom of the canonical quantum-classical disjunction, commonly expressed as a disconnect between theories applicable at small and large-scales.

41.) The prospect of a resolution is intimated *via* PM because the unit object in the PM lattice corresponds not to a local point particle state in $\mathbf{R}_4$ but to the fundamental coupling of properly-cancelling *pairs* of such states. Whereas the inertia-free limit case of SR is valid only as a proper state inside infinitesimal point domains inside GR, the equivalent inertia-free limit case inside PM is a scale-free principle properly valid over radial couplings (traced in SR and GR by null signal lines) at any and all scales, such that the gravitational mass shell of a PM "particle" lies at transformed (N-valued) unit radius from the particle's nodal origin (see para.13 *et seq*), and is interpermeable with the mass shells of all N other particles. One can see that in this scheme the mass shells of all particles are identically isomorphic to (and individually instantiate) the *holographic cosmic boundary* which is inflated $\mathbf{F}_2$, and they gravitationally interpenetrate one another at this distributed fractal boundary such that they both acquire and express gravitational mass *via* mutual nonlocal interaction on the radial phase of the PM space structure. In other words, the gravitational space structure *is* the structure of the "particles" that are "in" it, *is* their mass charge and also *is* their "Higgs coupling". The phase boundary between massless and massy particle phases is the internal boundary of a dual-phase metasymmetry which is shot through the entire spacetime structure and exists on all spacetime scales.

42.) At this point it will be useful to recap the quantum thermodynamical view of this metasymmetry as advanced in *Section 2.4*: In the emergence of real energy differences ΔU between systems (where a system is a measurement system comprising some interpermeating set of particle mass shells), real work is done so that in individual systems $\Delta U = Q - W \neq 0$, although $\Delta E_{set} = 0$. But according to PM the important point is that this work always represents the appearance of an *improper* distinction between two components of the internal energy - the heat energy ΔU and the free energy, F, the portion available for transformation to do external work. The emergence of this distinction corresponds to the *exclusively* ***improper*** *emergence* of 'electron mass' in thermodynamic disequilibrium, because the electron mass *is* this distinction, or $m_e = \Delta m = m_g - m_i$. But simultaneously the *exclusively* ***proper*** *cancellation* of this distinction in the null massless photon representation expresses the conservation of a supersymmetric equilibrium in each dyad.

43.) This emergent distinction of ΔU from ΔF is relativistically improper in the sense that it belongs not to the unit object (the PM dyad) but to the embedding set of systems constituting the environment on which work is done. Neither is it a property of that global embedding set inasmuch as there is no embedding meta-set constituting an external environment for it to do work on. In other words neither an individual dyad *AB* nor the universe as a whole has a real intrinsic mass because *both* are *closed* sets: The former minimal set is abstracted from its embedding, whilst the latter maximal set negates the meaning of embedding. One way of expressing this is to say that both these extremal sets are representations of *absolute* mass $|m|$, which is always always zero in the sense $m_g - m_i = 0$, whereas real relative mass Δm belongs to embedded multiplets where $m_g - m_i \neq 0$. If we think about this we understand in a new way how the *essence* of mass in relativity is in fact its pluralistic nonlocality.

44.) And now we should again compare our proposal with the Mach-Weber theory of inertia for a universe of particles interconnected by nonlocal far-actions, as introduced in *Section 1.1.*. Given the postulate that *the sum of all forces on a particle is zero in all coordinate frames*, Machian inertia arises from Weber's force law

$$F_{1,2} = -H_g \frac{m_1 m_2}{r^2}\left[1 - \frac{6}{c^2}\left(\frac{\dot{r}^2}{2} - r\ddot{r}\right)\right] \qquad (33)$$

which modifies Newtonian gravitation by terms proportional to the relative velocity and acceleration. Here the inertia is a dynamical reaction force. Analyses by Assis (1993) and Assis & Graneau (1994,1995) show that the long-range $1/r$ force term proportional to the acceleration implements Mach's principle by effectively dividing the cosmic mass distribution into isotropic and anisotropic components. The long-range $1/r$ force locally will be dominated by the isotropic gravity of the 'fixed galaxies', generating inertia as a dynamical reaction against the $1/r^2$ Newtonian accelerations produced by anisotropic nearby masses. An especially interesting result of the Weber force law is one echoed in PM, and that is that in general the effective inertial mass of a body will not be isotropic, and will depend on the potential where the body is located.

45.) According to Weber's postulate inertial force is the dynamical reaction force due to the 'fixed galaxies' that restores the vector sum of forces on a particle with gravitational mass m_g to zero for all observers. We accept the spirit of this principle, so that

$$- \overline{F} = m_i \overline{a} = - m_g \overline{a} \tag{34}$$

Conventionally the sign is the property of the force vector and therefore vanishes when acceleration goes to zero, to leave just the identity

$$m_i \equiv m_g \tag{35}$$

which states the equivalence principle, a simultaneous identity of two indiscernible scalars which is an *unnatural relation* in Newtonian physics and an unrealised *identity in principle* in GR. But in our theory there is no longer an unnatural relation nor an identity in principle, but instead a conditional natural relation,

$$\vec{m}_i - \overleftarrow{m}_g = 0 \tag{36}$$

because m_i and m_g are force *operators* and the total mass-energy is always the *zero sum* of two interrotateable antiparallel vector operations. Likewise $+t$ and $-t$ are interrotateable by a congruent transformation of the line into itself. Thus

$$- \Delta t_{m_i} + \Delta t_{m_g} = 0 \tag{37}$$

which identifies the null identity of m_i and m_g as a *proper* characteristic of the special class of lightlike zero vectors but *not* an *improper* characteristic of the classes of positive timelike or spacelike vectors.

46.) Where these two equal and opposite antiparallel vectors ***m*** are interrotateable we can't identify one as an active force and the other as a reaction. This means that the effect of substituting inertial mass $-m_i$ for one of the gravitational masses m_g in the expression for the gravitational force

$$F = - G \frac{m_1 m_2}{r^2} \tag{38}$$

leads to a sum of four different possible orderings

$$\overline{F} = G\left(\frac{-\vec{m}_1 \cdot \vec{m}_2}{r^2} + \frac{\vec{m}_1 \cdot \vec{m}_2}{r^2} + \frac{\vec{m}_2 \cdot -\vec{m}_1}{r^2} + \frac{-\vec{m}_2 \cdot -\vec{m}_1}{r^2} \right) \tag{39}$$

which for interchangeable magnitudes of *m* reduces with *r* = *unity* to

$$\overline{F} = G\left(-\vec{m}^2 + \vec{m}^2\right) = 0 \tag{40}$$

giving us zero intrinsic gravitational force between two points of measurement (elementary particle positions).

47.) Whilst this would seem rather a startling claim in the context either of Newtonian or of GR gravitation, it is quite trivial in the PM gravitational structure. It can't be emphasised often enough that the null mass vector *must not be confused with annulment of the scalar mass that one says couples to the universal gravitational field in GR*. This scalar mass corresponds to the coupling at the boundary of the unit-radius mass shell in PM, which is itself a scale-free *phase* of the PM space structure and this phase is not extricable. We say only that an idealised state of equilibrium for a system *conventionally* regarded as 'two particles' (a pair of PM nodes or graph vertices) is that *m* vanishes on the lightlike path between them. This is *not* the same as saying that this system feels no gravity or inertia, of course, because the conditions of 'feeling' gravity and inertia are precisely those which destroy the equilibrium. We merely mean that, because the particle mass shell lies at unit proper radius from the point of measurement, in an abstracted system of two measurement points the radius of each mass shell would therefore be single-valued (unit radius) and each position would lie at unit distance from the other. Or, a gravitational potential only arises because in a system of *N* particles the mass shell radii become *N*-valued.

48.) It's helpful to characterise the proper null force of Eq 40 as a zero-point vacuum potential. From this point of view, if we *could* perfectly isolate the two nodes (in a thought experiment) then we would be excluding all of the *N* - 1 external radial modes of the PM inertial structure. Analytically, the fundamental wave mode of this isolated unit object represents a zero-point of energy, but has an equivalent representation as the *synthesis* of an indefinite number of partial modes. In the absence of any internal constraint (other than the bounding pair of nodes at unit distance) the number of wave modes is infinite, which would imply an infinite vacuum energy and an ultraviolet-catasrophic overpressure - analogous to an infinite Casimir force - except that these are purely virtual modes of the unrealised unit object. In PM the actual space structure is what both elicits *and* simultaneously limits real modes of the unit object. Our foundational *exclusion principle* can be expressed in these terms: The vacuum energy of the internal partial wave modes between any two vertices is *equal and opposite* to the gravitational energy of the set of all the external modes co-original at the vertices (which is our incorporation of Weber's principle of zero net force on the unit object - summed over *pairs* of position states), and therefore corresponds to an imaginary inflationary energy for which the constraint co-emergent with $\mathbf{R}_4$ supplies a gauge that we call gravitation, and on which the same constraint therefore sets both upper and lower bounds. In the real metrical space of $\mathbf{R}_4$ where node number proliferates in the limit of N, we can think of this vacuum potential as being transformed down approximately in proportion to $1/N$ where N is very large, but nowhere to real zero. The zero-point null force inside (abstracted) dyad*s is an unrealisable limit for any improper (transverse) measurement operation*, because real operations are only possible on the subnet phase of the space structure where $N = 3$ or more. Thus we predict that a residual vacuum pressure occurs on these elementary triadic structures (instantiating flat $\mathbf{F}_2$ on all real scales), which we can think of as a *repulsive* force phase inside $\mathbf{R}_4$ which is not perfectly cancelled by the gravitational/inertial mass constraint that appears as *attraction* over systems of large N (analysable to an interconnectivity of large numbers of triads).

49.) The substitution we have made to arrive at Eq 40 would be just as meaningless in a Mach-Weber theory as in Newton's or in GR. The $1/r$ force term which implements Mach-Weber inertia is introduced as a long-range 'correction' to Newtonian gravity assuming a space of central forces in which the distribution of particles varies in absolute Newtonian time. Although the gravitational far-action is nonlocal in this theory, particle mass remains a scalar coupling to a gravitational potential which is assumed *monopolar* and *attractive* with t positive. But this can be seen as perpetuating a general contradiction in Newtonian mechanics, where for purposes of stating the causal relations between consecutive configurations of large scale systems a single and uniform direction of time is assumed; yet in the equations of mechanics t is only an independent variable which can be changed to $-t$ without altering anything. Newtonian mechanics is perfectly time-reversible. Why is Newtonian gravity not?

50.) It is striking that Newtonian physics enshrines two features lately 'rediscovered' as quantum principles: An essential reversible symmetry and action-at-a-distance.[89] It is commonly assumed

89 Newton's conception of action-at-a-distance was not strictly nonlocality in the modern sense, of course, since Newton could not have understood the relativistic basis of its inverse, locality.

that t in the classical laws of motion is an idealised abstraction which has no real connection to the world of experience. Yet this point of view itself sits uncomfortably with quantum theory, where time-reversed particle trajectories have a formal equivalence and a physical meaning. Eq.40 thus seems an appropriate correction to the classical case, describing a potential F_g **:** $-F_g$ dipole with an underlying true time-reversal symmetry which appears repaired in a nonlocal (radial) phase of PM space but gets spontaneously broken locally (transversely).

51.) To summarise: PM proposes that on the 'particle scale' inertial and gravitational mass are mirrors of one another. (Remember that this relates not to a quantity in our system but to a quality - a proper, radial pairing of real position states, or only to what might be called lightlike connections or 'longitudinal field components'). Nonlocally cancelled null time +/-t gives the sum of +m and -m which gives the photon case, in which Newtonian scalar quantities m_1 and m_2 would become degenerate states. But in the PM representation the degeneracy is averted (paradoxically) by the fact that there never were two quantities in the first place; rather there are (interrotateably) positive and negative vectorial representations of the *same* quantity m_i *(inertial)* and m_g *(gravitational)* polarised by exclusionary *unit scale*, commutative between (-m)·(m) and (m)·(-m), which transforms as a time (like the total energy in SR where the invariant is 'rest energy'). This is not the case for 'many-particle' systems in general, where $m_1 \,.\, m_2$ does apply and there is no equivalence.

52.) A lightlike radial null-t, null-m, s^2=0 symmetry breaks to timelike/spacelike transverse construction of s^2 = *positive*-selected timelike invariant intervals. At any point of measurement lightlike vectors are exclusively those longitudinal components associated with the radiation part of the supersymmetric 'field'. The longitudinal forces proportional to mass that vanish along these components vanish altogether for the vertex which is their common origin. The commonplace reflection of this in relativistic particle physics is that a photon connecting an atom in your retina with the photosphere of a distant star has zero rest mass and does not 'know' time, or: Interactions between charges occur only where the relativistic interval s^2 vanishes. The 'cosmological time' attached to this scale-free relation is in fact imaginary, and its real projection occurs in the transverse 'scattering' angles that construct spacetime out of the non-vanishing *im*proper intervals of timelike and spacelike vectors.

53.) To put this another way, if *all* intervals were *co-original* radii (i.e., if the only field components were longitudinal ones) then we would say that there was an indiscernibly identical degeneracy in all position states in a PM space of infinite radius; there would obviously be neither real space nor time nor mass. The fact that there are *many non*-degenerate origins of radial field components involves the corollary that each radius vector is only conjoint at two origins, say *A* and *B*; at origin *A* and at origin *B* the vast majority of intervals are transverse and have only improper states. It is the angular generation of plural space out of singular degeneracy by these transverse components that, by its control of the *differential* dispositions of radial nulls over the skies of *A* and *B* (alike or near-twinned superpositions if close, unalike if remote), generates the deformation - *rotation* or *curvature* - in the unit vector *AB* which we call a state of non-zero mass-energy.

54.) Gravitation can be seen as a locally attractive real field due to a nonlocally repulsive virtual force. The *vector* sum of all mass-generators is zero at any single *x,* where the complex null symmetry is unbroken, but the symmetry is broken to real values for *plural x*'s, because in many-centred space not all zeros at all origins are interrotateable. The breaking of this symmetry gives a *gradient* of zero-point states, representable as an affine tensor field of tangent vector spaces as in GR. The vacuum rigidity (virtual repulsive "force") between two origins *A* and *B* can be seen as the complex inflaton mode of PM space. PM inflation occurs on a scale-free phase of $\mathbf{R}_4$ at all epochs and is the transformation of unit scale to all possible real scales.

55.) The contribution of each remote vertex to the gravitational/inertial mass constraint will be proportional to the cosine of the real angle at the vertex, so that field potentials vary although the linear force is independent of scale. In terms of the underlying complex vector representation the phase angle only, and not the amplitude, enters the gravitational component of local force at a given vertex. On the complex quantum phase of the space structure where all radii are orthogonal (i.e., on $\mathbf{F}_2$) the calculation of the mass field is sensitive only to *numbers* of terms that contribute identically, i.e. it is a scalar field. But in $\mathbf{R}_4$ the emergence of improper transverse terms controls the angular distribution of proper radial terms in a vector field. So the dipolar resultant of unit vacuum force measured across a pair of points *y,y* or *z,z* bounding unit scale becomes a decreasing attractive potential with increasing real transverse distance *away* from *z*, like

$$<<w \quad <x \quad y \quad >z \; z< \quad y \quad z> \quad w>>$$

or, looked at in another way, a repulsive potential reduces *towards z***,** which produces 1/r gravity in the embedding context of a global expansion.

56.) In a nutshell: the quantum basis state of gravitation is the zero-point unit scale (representing the PM exclusion principle or the non-degeneracy of position states) and its modulus amplitude is determined by a hypercomplex calculation that proceeds (as it were in pseudo-time) over the whole system of interpenetrating orthonormal unit-radius mass shells vertex by vertex. The non-equivalence of real and imaginary magnitudes in $\mathbf{R}_4$ (+*t* and -*t*) and in quantum mechanical state space (+/-*x*, +/-*i*) is a representation of an emergent irreversibility in the ensemble. The emergence of transverse phases in the ensemble breaks a time-free, complex-valued, nonlocally-coupled, anyonic supersymmetry into intervallic modes having non-equivalent antiparallel vector representations (and generates local coupling of fermions by bosons). The non-equivalent representations separated in the ensemble are technically real and imaginary, +*t* and -*t*. A global time sign (+ by convention) emerges as the statistical residual of an ensemble of periodic non-cancelling transformation processes performed by the ensemble on itself, and *N* individually-cancelling antiparallel vacuum vectors acquire directed real values donated to themselves *via* the ensemble: The *N* different zero-point complex vacua (scale-free "quanta of the inflaton field") generate non-zero real residual vacuum vectors that correspond (improperly conceived) to imperfect cancellations of kinetic and potential energy. This ensemble of vacuum defects is thus to be the origin of a non-singular real-valued action potential which, when characterised as a negative potential energy equal and opposite to a positive kinetic energy, defines the gravitational action potential of spacetime.

57.) This *potential* is a dipole. Both poles of the dipole are inherently pluralistic system properties. If any pair of nodes could perform idealised 'measurements' on one another directly as though the rest of the graph did not exist they would measure no proper gravitational or inflational potential along the direction between them, which is to say that they would behave always as though the potential - a distance - were zero, and this is what we say two electrons do when we define their charge coupling on the photon null signal line where s^2 is zero. But such an idealised condition is antithetical to the physical meaning of measurement and this fact is embodied in the causal structure of $\mathbf{R}_4$ which ensures that $s^2 = 0$ is not available to objects with mass, or equivalently that $m = 0$ is not available to particle pairs coupled transversely into the complete graph of PM space, or in yet other terms: the condition $m_g = 0$ for fermions identifies only a causally decoupled subspace of $\mathbf{R}_4$.